'Low-dimensional magnetism' by Diptiman Sen

Note Title 27-11-2018

Ferromagnets & Antiferromagnets

$$\underset{1}{\cdot}\underbrace{\quad\quad}_{a}\underset{2}{\cdot}\quad\underset{3}{\cdot}\quad\underset{4}{\cdot}\quad\cdot\quad\cdot \qquad \xrightarrow{\quad} n$$

$$\vec{S}_n^{\,2} = S(S+1)\hbar^2$$

$$H = -J\sum_n \vec{S}_n\cdot\vec{S}_{n+1}$$

$J > 0$

Ferromagnetic system

$$\uparrow\uparrow\uparrow\uparrow\uparrow$$

All $S_{nz} = S\hat{z}$

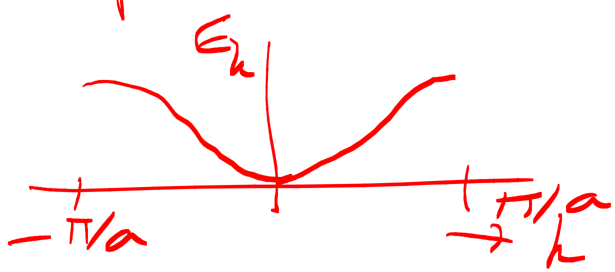
Spin-wave theory:

Excited state with momentum k

$$|k\rangle = \sum_n e^{ikn} \, | \underset{S}{\uparrow} \underset{S}{\uparrow} \underset{S}{\uparrow} \cdots \underset{S-1}{\overset{}{\underset{n}{\uparrow}}} \underset{S}{\uparrow} \underset{S}{\uparrow} \underset{S}{\uparrow} \underset{S}{\uparrow} \rangle$$

$$E_k = 2JS\,(1 - \cos(ka))$$

Spin-wave (magnon) dispersion



Gapless at $k=0$

Around $k=0$,

$E_k \sim k^2$

Antiferromagnet

$$H = J \sum_n \vec{S}_n \cdot \vec{S}_{n+1}$$

$$J > 0$$

$$\uparrow \quad \downarrow \quad \uparrow \quad \downarrow \quad \uparrow \quad \downarrow$$

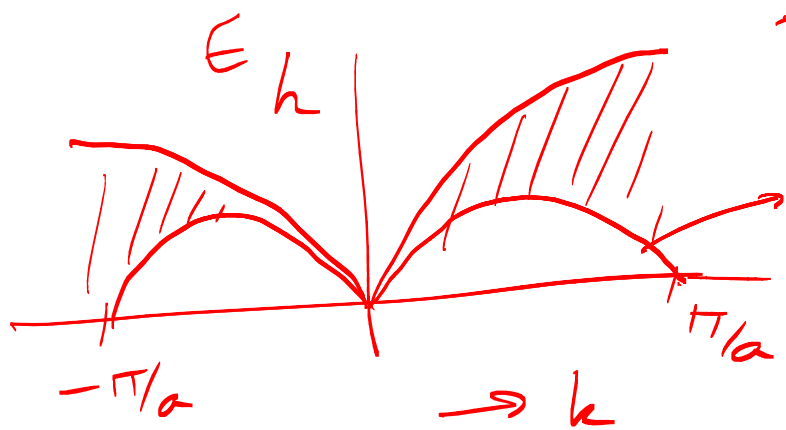
Neel configuration

$S_{1z} = +S \qquad S_{2z} = -S \qquad S_{3z} = +S$

Bethe ansatz : Ground state has total spin $= 0$ (singlet)

Lowest excited state ($S = 0$ and 1)

Spin-$\frac{1}{2}$ AFM chain



$\pi J \sin\left(\frac{ka}{2}\right)$

$\frac{\pi J}{2} \sin(ka)$

Gapless at $k = 0$ and $\pm \pi/a$

Dispersion is linear at $k = 0, \pi$

$S = 1, 3/2, \ldots\ldots ?$

$S \to \infty$ Semiclassical limit

Anderson, Phys Rev. 86, 694 (1952)

$$\begin{array}{cccc} \uparrow & \downarrow & \uparrow & \downarrow \\ S & -S & S & -S \\ n=1 & n=2 & n=3 & \end{array}$$

Holstein-Primakoff transf

$n=1$:

$$S_{nz} = S - a_n^\dagger a_n$$

$$S_{n+} \cong \sqrt{2S}\, a_n, \quad S_{n-} \cong \sqrt{2S}\, a_n^\dagger$$

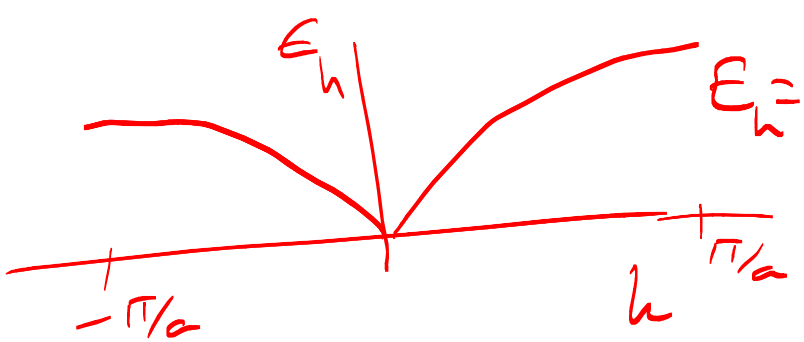
$n=2$:

$$S_{nz} = -S + a_n^\dagger a_n$$

$$S_{n+} \simeq \sqrt{2S}\, a_n^\dagger, \quad S_{n-} \simeq \sqrt{2S}\, a_n$$

Write H in terms of $a_n, a_n^\dagger$ up to second order.

$$\sim a^\dagger a, \quad a^\dagger a^\dagger, \quad aa$$

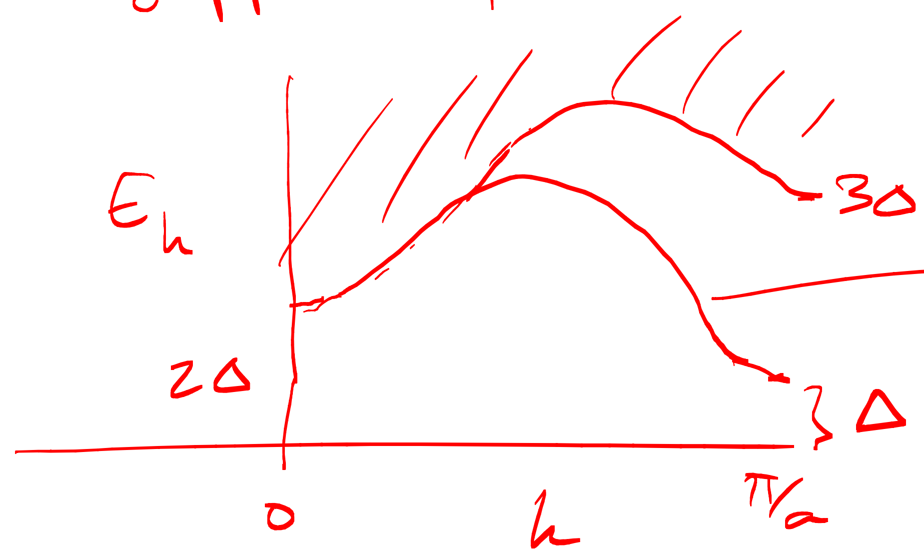


$$E_k = 2JS \sin|ka|$$

Haldane (1981-83) that the spectrum is gapless only if $S = 1/2, 3/2, 5/2, \ldots$

and is gapped if $S = 1, 2, 3, \ldots \rightarrow$ gnd. state has $S=0$ and is non-degenerate

$$H = J \sum \vec{S}_i \cdot \vec{S}_{i+1}$$



isolated curve

$\Delta = 0.41\,J$

Excited states $S=1$.

DMRG

For general integer S,

$$\Delta \sim e^{-\pi S} \qquad \text{(Haldane, Affleck, ...)}$$

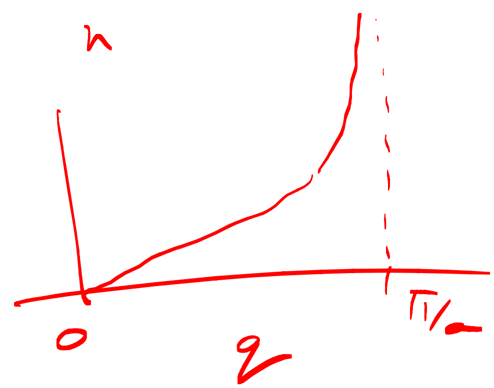
$$\to 0 \quad \text{as} \quad S \to \infty .$$

Structure function: $S(q)$

$$\underbrace{\langle \vec{S}_0 \cdot \vec{S}_n \rangle}_{\text{ground state}} \sim \frac{(-1)^n}{|n|} \sqrt{\ln |n|} \qquad S = \frac{1}{2}$$

$$(-1)^n \to e^{i\pi n}$$

$$S(q) = \sum_n e^{iqna} \langle \vec{S}_0 \cdot \vec{S}_n \rangle_{\text{gnd. state}}$$



$S(q) \to \infty$ as $q \to \pi/a$

$\sum_n \vec{S}_n = 0$ in gnd. state.

$S=1$: $\langle \vec{S}_0 \cdot \vec{S}_n \rangle \sim \frac{(-1)^n}{|n|} e^{-n/\xi}$

$\xi \sim \frac{1}{\Delta}$

$\xi =$ correlation length $\simeq 6$

$S(q)$

Frustrated model:

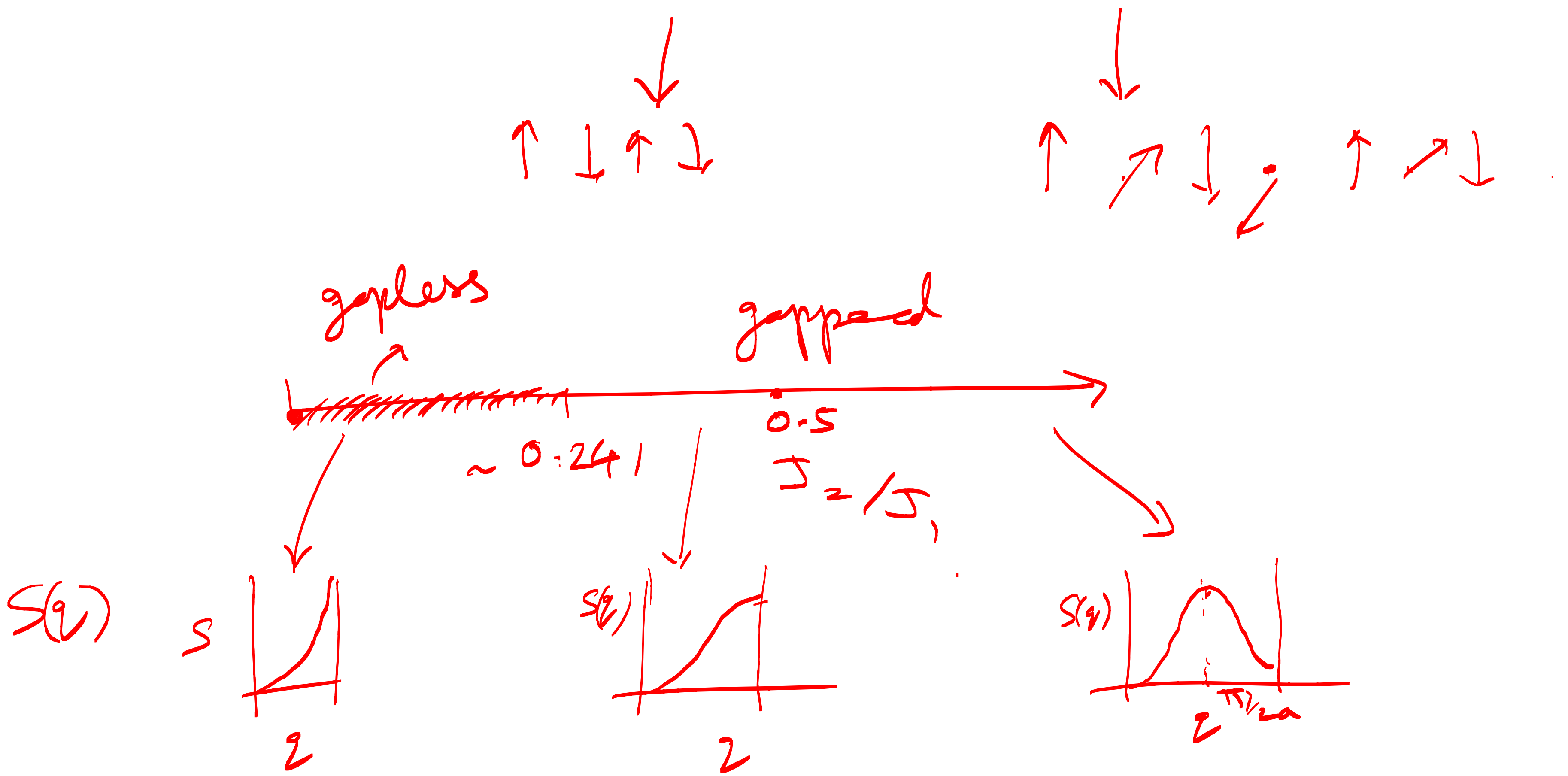
$$H = J_1 \sum_n \vec{S}_n \cdot \vec{S}_{n+1} + J_2 \sum_n \vec{S}_n \cdot \vec{S}_{n+2}$$

$J_1,\ J_2 > 0$

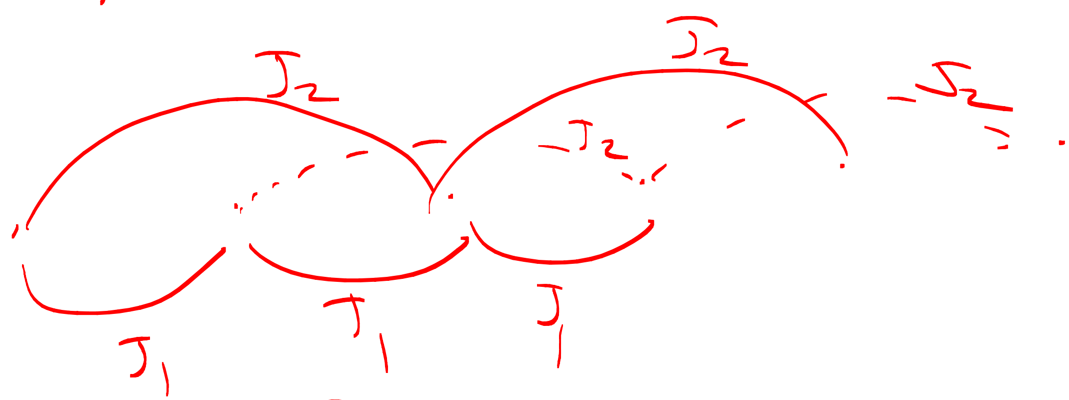
spin-$\frac{1}{2}$ model

$J_2 \to \infty$:

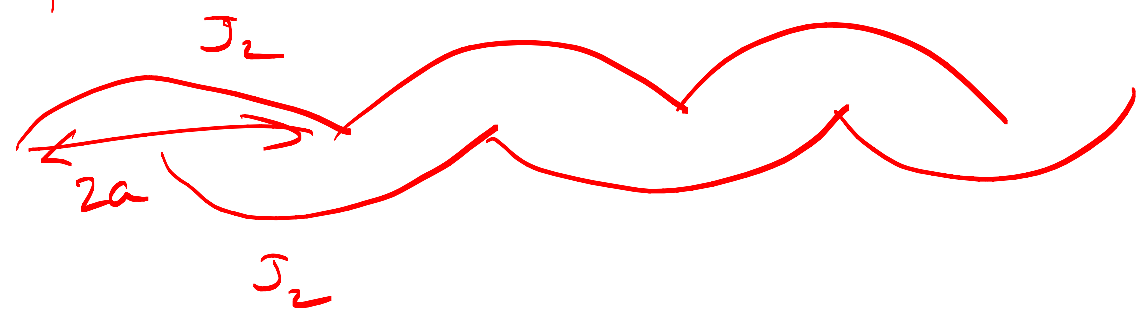
Two decoupled chains

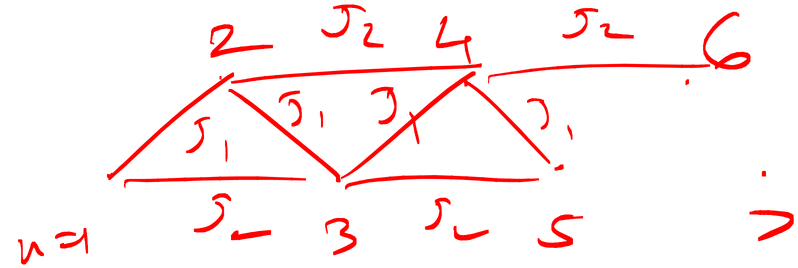

gapless
gapped
~ 0.241
0.5
J2/J1
S(q)
S
q
S(q)
q
S(q)
q
π/2a

$$H = J_1 \sum_n \vec{S}_n \cdot \vec{S}_{n+1} + J_2 \sum_n \vec{S}_n \cdot \vec{S}_{n+2}$$



$J_1 = 0$





$J_2/J_1 = 1/2$: Majumdar-Ghosh model

$$H = J_1 \left[\sum_n \vec{S}_n \cdot \vec{S}_{n+1} + \frac{1}{2} \sum_n \vec{S}_n \cdot \vec{S}_{n+2} \right]$$

$$= \frac{J_1}{2} \sum_n \left(\vec{S}_n + \vec{S}_{n+1} + \vec{S}_{n+2} \right)^2 + \text{constant.}$$

Gnd. state is a product of singlets over nearest neighbors

$n=1 \quad n=2 \quad n=3 \quad n=4 \quad n=5 \quad n=6$

$$\prod_n \frac{1}{\sqrt{2}}\left(|\uparrow_1 \downarrow_2\rangle - |\downarrow_1 \uparrow_2\rangle \right)$$

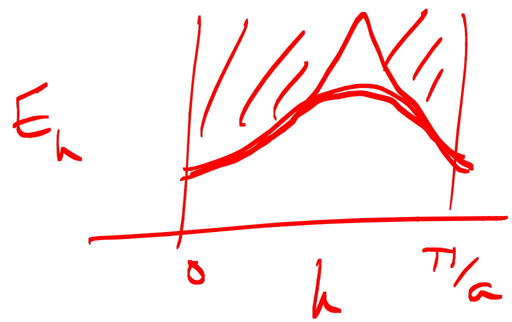
Gnd. state

$S=0$

$n=0 \quad n=1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \ldots$

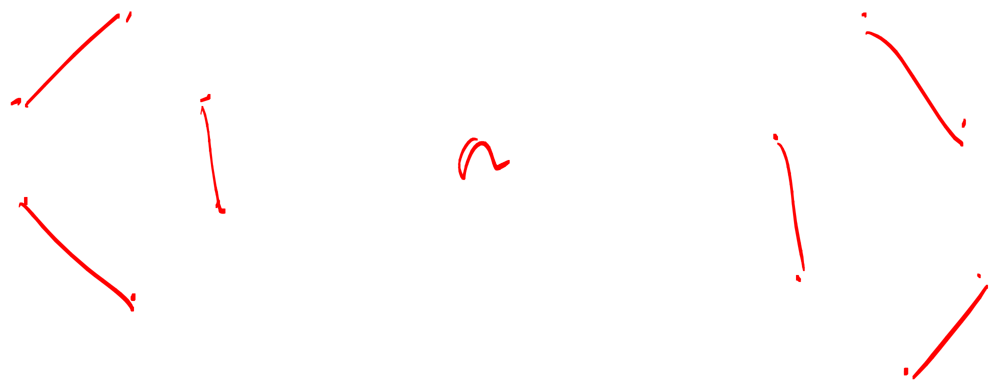
valence bonds

$$\rightarrow \prod_n \frac{1}{\sqrt{2}}\left(|\uparrow_0 \downarrow_1\rangle - |\downarrow_0 \uparrow_1\rangle \right)$$



Shastry & Sutherland,
Phys. Rev. Lett.
47, 964 (1981).

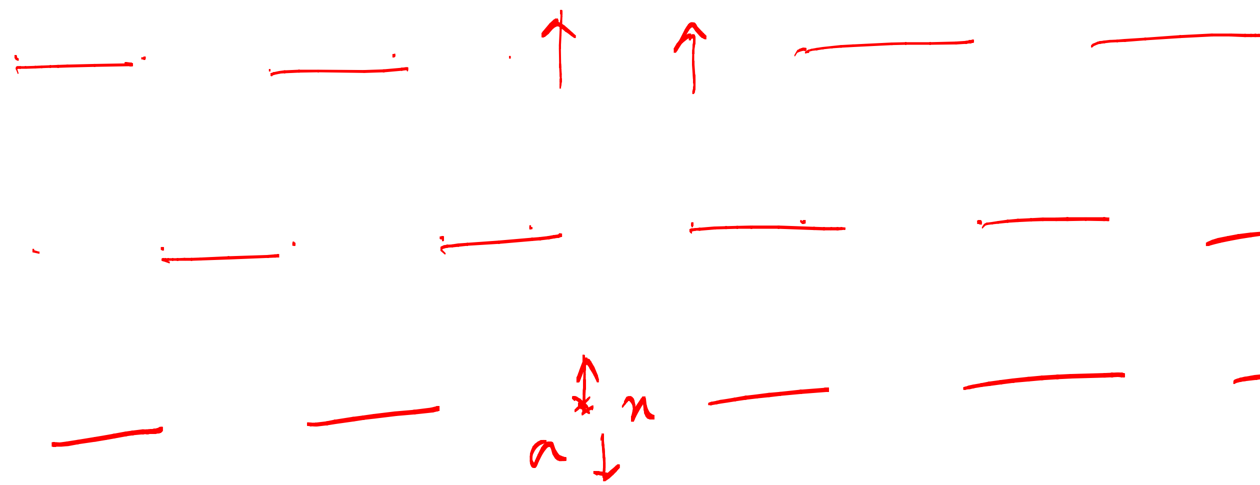
Benzene



$|I\rangle =$

$|II\rangle =$

Soliton
domain wall



$\rightarrow |k\rangle = \sum_n e^{ikn} |n\rangle$

Lowest possible excitation

$|I\rangle$ on the left

$|II\rangle$ on the right

Spinon

Dimerisation $\qquad CuGeO_3 \qquad (Cu^{2+} = \text{Spin-}\tfrac{1}{2})$

$\delta \simeq 0.06$

$\vec{S}_1 \quad \vec{S}_2 \quad \vec{S}_3 \quad S_4 \quad \cdot \quad \cdot \quad \rightarrow$ Peierls instability

$J_1(1+\delta) \quad J_1(1-\delta) \quad J_1(1+\delta) \quad J_1(1-\delta)$

$$H = J_1(1+\delta)\,\vec{S}_1\cdot\vec{S}_2 + J_1(1-\delta)\,\vec{S}_2\cdot\vec{S}_3 + \ldots$$

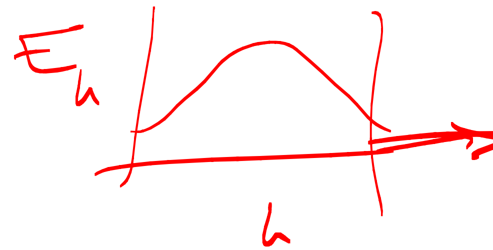
$\delta \to 1: \qquad 2J_1 \quad 0 \quad 2J_1 \quad 0$

Gnd. state is a singlet and non-degenerate

δ

1 $\longrightarrow$ gapped

0 $\longrightarrow$ gapless point



$\Delta \sim |\delta|^{2/3}$

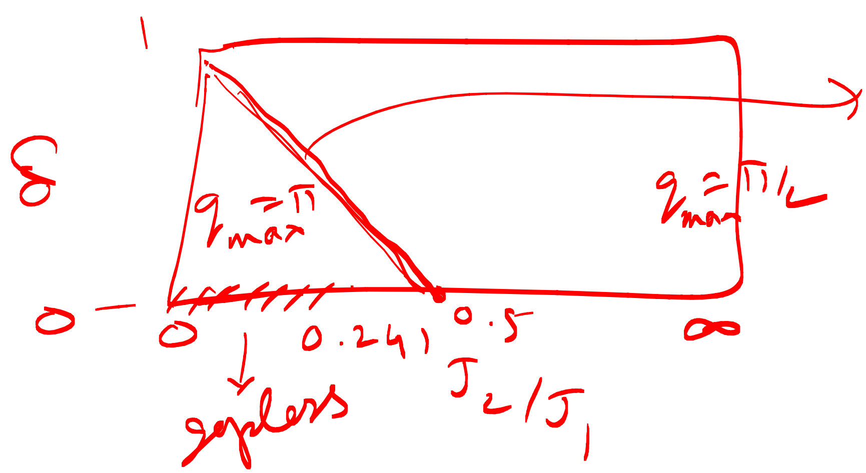
$$\chi \sim e^{-\Delta/k_B T} \text{ as } T \to 0$$

$\sim$ constant as $T \to 0$

for gapless systems.

$$H = J_1 \sum_n \left[1 + (-1)^n \delta\right] \vec{S}_n \cdot \vec{S}_{n+1}$$

$$+ J_2 \sum_n \vec{S}_n \cdot \vec{S}_{n+2}$$



$$\frac{2J_2}{J_1} + \delta = 1$$

$\rightarrow$ gnd. state exactly solvable

$S(q) \rightarrow q_{max}$

Magnetisation plateaus

2

1 3

$$H = J_1 \left(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot S_3 \right)$$

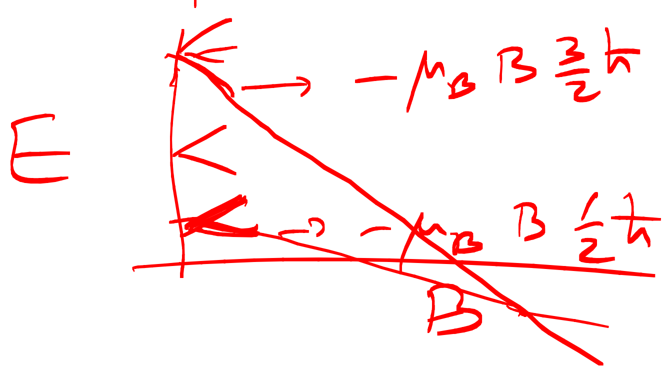
$J_1 > 0$

$S = 3/2$

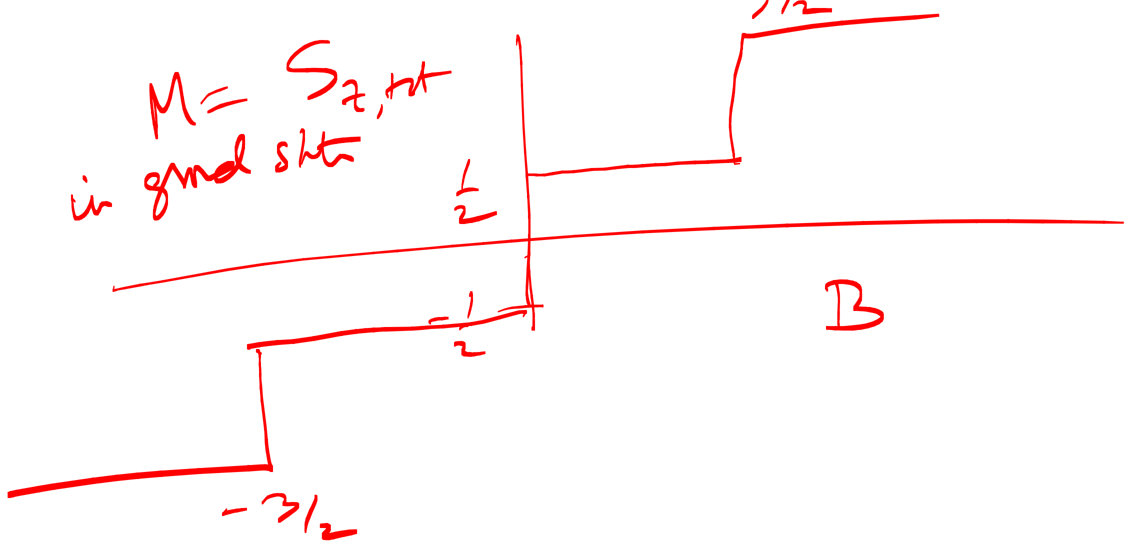
$S = 1/2$

$S = 1/2$

$$H = J_1 \left(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3 \right) - \mu_B B \left(S_{1z} + S_{2z} + S_{3z} \right)$$

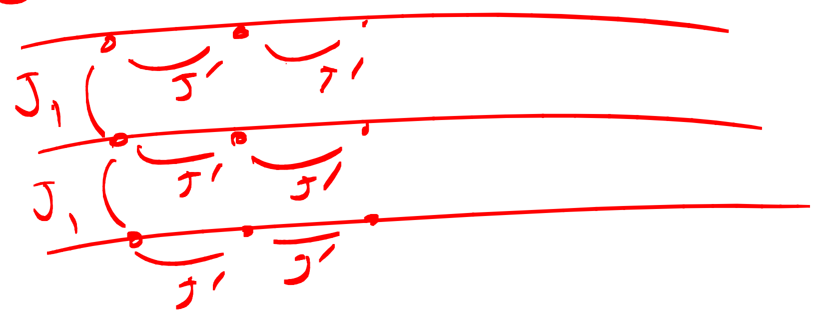


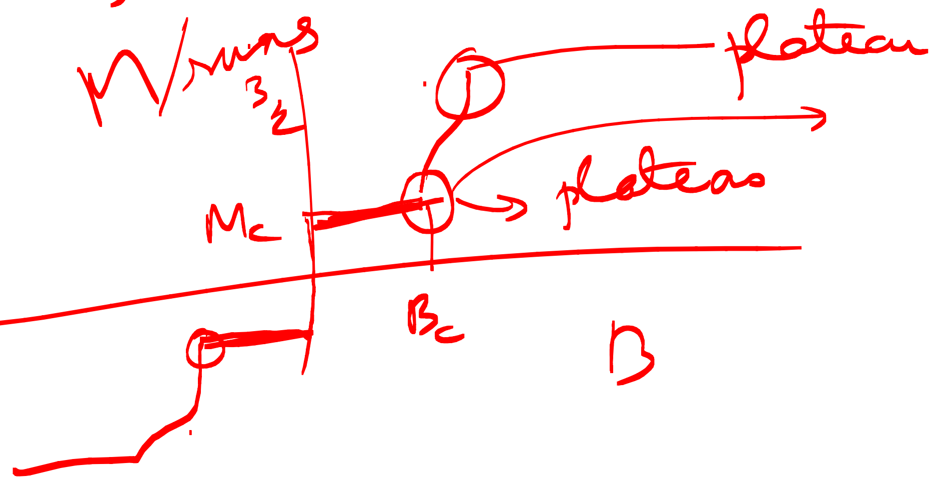
Above some $B_{critical}$, $S = 3/2$, $S_z = 3/2$ becomes the ground state



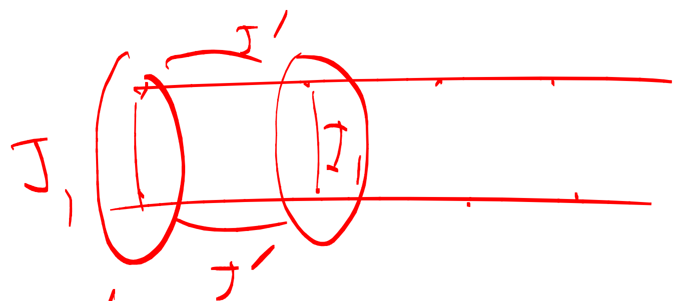
Cabra, Honecker and Pujol, Phys. Rev. Lett. 79, 5126 (1977).

3-leg ladder

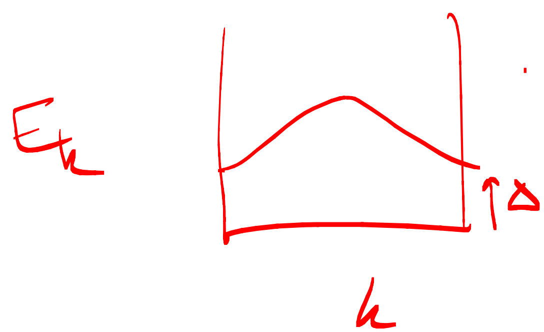




$M - M_c \sim \sqrt{(B - B_c)}$

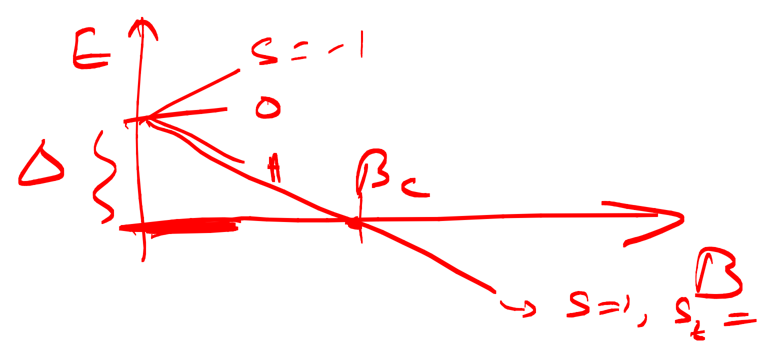


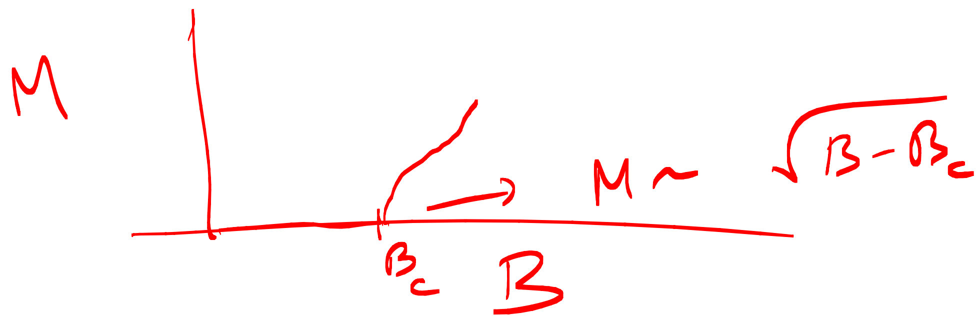
$S=0$ or 1 $\rightarrow$ integer spin $\rightarrow$ Haldane gap



ground state $\Rightarrow S=0$

Excited state $\Rightarrow S=1$





M
B_c
B
$M \sim \sqrt{B - B_c}$