Note Title 26-11-2018

Indo - French Meeting on Molecular Magnetic Systems

Fundamentals of Magnetism

L in a stationary frame

• Stationary observer

L

v

$$L' = \frac{L}{\gamma} \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

c is the velocity of light

Velocity of light is a constant

$$c = \frac{1}{\sqrt{\mu \epsilon}}$$ ⟸ Maxwell's equation

μ and ϵ do not depend upon the frame of reference

c must be a constant in all frames of reference

→ Carrying current

→ drift velocity of negative charges is v_d

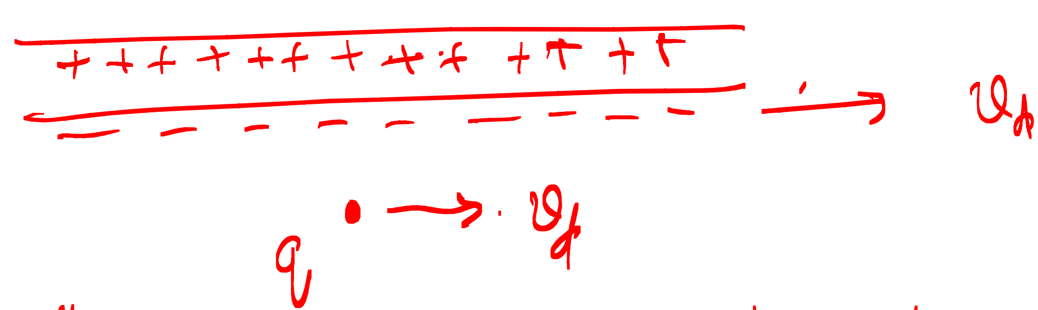
q •

stationary

$$\rho_+ = \rho_-$$

ρ_+ and ρ_- are linear densities of +ve and negative charges

$$\rho_+ = Q_+/\ell \qquad \rho_- = Q_-/\ell$$



$-$ve charges in the wire appear stationary for the moving charge while the positive charges appear to be moving with velocity v_d

$$\rho_-' = \frac{Q^-}{L} \qquad \rho_+' = \frac{Q^+}{L'} \qquad L' = L\sqrt{1 - v^2/c^2}$$

Since $L' < L$ $\quad \rho_+' > \rho_-'$

This leads to an attraction of the electron by the wire

This attractive force is the magnetic force

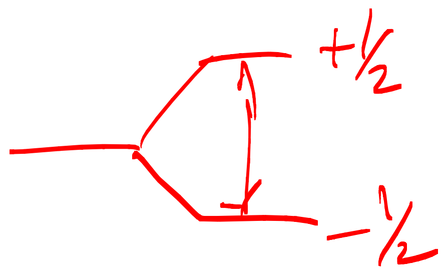
Lorentz force on a moving charge q with velocity $\vec{v}$ is given

$$\vec{F} = q\vec{E} + \frac{q}{c}\vec{v} \times \vec{B}$$

$\vec{B}$ is the magnetic field and $\vec{E}$ is the electric field

magnetic force is very weak 1 Tesla of magnetic field (10^4 gauss)

electron in a magnetic field of 1 Tesla the splitting of the spin states



$\Delta E \sim k_B T_n \qquad T_n\,(1\,\text{Tesla}) \sim 1.4^\circ K$

$\sim 1\,\text{cm}^{-1}$

there is a current due to the movement of electron around the proton

$$I = \frac{e}{\tau}$$

τ is the time taken for one period

$$\tau = \frac{2\pi r}{v}$$

$$I = \frac{e\,v}{2\pi r}$$

Magnetic moment is $\vec{\mu} = I\vec{A}$, $\vec{A}$ is the area vector and I is the current.

$$= \frac{ev}{2\pi r} \cdot \pi r^2 = \frac{e}{2} vr = \frac{e}{2m} mvr$$

$$= \frac{e\hbar}{2m} = \frac{eh}{4\pi m}$$

Bohr Magneton $\frac{eh}{4\pi mc}$

Associated with every angular momentum we have a magnetic moment

$L \propto M$ and the constant of proportionality is called the gyromagnetic ratio

$M = gL$ For orbital angular momentum we have

$$g_L = 1$$

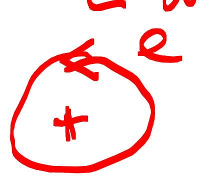
But for spin angular momentum of the electron

$$g_S = 2$$

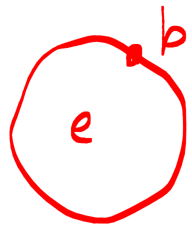
Both orbital and spin angular momenta then what happen to g

$$M_{\text{Landé}} = (g_L L + g_S S) = (L + 2S)$$

When both L and S are nonzero, there is spin orbit interaction



taken the proton as at rest and electron as moving



Magnetic field produced by the proton

$$\vec{H} = \vec{E} \times \vec{V}$$

$\vec{E}$ is the electric field and $\vec{V}$ is the velocity

$$\vec{E}' = -\vec{\nabla} U \qquad U = \frac{e^2}{r}$$

$$\vec{E}' = \frac{e^2}{r^2}\hat{r}$$

$$\vec{H} = \vec{E} \times \vec{V} = \frac{e^2}{r^2}\,\hat{r} \times \vec{V}$$

$$= \frac{e^2}{m r^3}\left(m\vec{r} \times \vec{V}\right)$$

where $\hat{r}$ is the unit vector $\vec{r} = r\hat{r}$

$$\vec{H} = \frac{e^2}{mr^3}(\vec{r} \times \vec{p}) = \frac{e^2}{mr^3}\vec{L}$$

Spin orbit interaction $= \vec{H} \cdot \vec{S} = \frac{e^2}{mr^3} \vec{L} \cdot \vec{S}$

$\vec{L} \cdot \vec{S}$ is in the Hamiltonian

then the quantum number that is given for the total angular momentum is $J = L+S, \ldots |L-S|$

$\vec{M} = (\vec{L} + 2\vec{S}) = g_J \vec{J}$ $\quad g_J$ is the g factor for the state J

$(\vec{L}+2\vec{S})$ and $\vec{J}$ transform similarly

$$\langle \vec{L}+2\vec{S} \rangle = g_J \langle \vec{J} \rangle$$

$$\langle (\vec{L}+2\vec{S})\cdot\vec{J} \rangle = g_J \langle \vec{J}\cdot\vec{J} \rangle = g_J \langle J^2 \rangle$$
$$= g_J J(J+1)$$

$$\langle (\vec{L}+2\vec{S})\cdot(\vec{L}+\vec{S}) \rangle = \langle L^2 + 3\vec{L}\cdot\vec{S} + 2S^2 \rangle$$
$$= L(L+1) + 2S(S+1) + 3\vec{L}\cdot\vec{S}$$

$$\vec{L}\cdot\vec{S} = \left((L+S)^2 - L^2 - S^2\right)/2 = \left(J^2 - L^2 - S^2\right)/2$$

$$\langle (L+2S)\cdot \vec{J} \rangle = L(L+1) + 2S(S+1) + \frac{3}{2}\left(J(J+1) - L(L+1) - S(S+1) \right)$$

$$= \frac{3}{2} J(J+1) - \frac{L(L+1)}{2} + \frac{S(S+1)}{2} = g_J J(J+1)$$

$$g_J = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}$$

$$\mathcal{H}_{mag} = \mu_B(\vec{L} + 2\vec{S})\cdot \vec{H}$$

$$\mathcal{H}_{mag} = \mu_B(\vec{L} + 2\vec{S})\cdot \vec{H} + \lambda \vec{L}\cdot\vec{S}$$

A perturbation calculation for energy

$$E = E^{(1)} + E^{(2)} + \cdots$$

$$E^{(1)} = \mu_B \langle \Psi_g | (\vec{L} + 2\vec{S}) | \psi_g \rangle \cdot \vec{H} + \langle \psi_g | \lambda (\vec{L} \cdot \vec{S}) | \psi_g \rangle$$

$$E^{(2)} = \mu_B \frac{\langle \Psi_g | (\vec{L} + 2\vec{S}) \cdot \vec{H} | \psi_e \rangle \langle \psi_e | (\vec{L} + 2\vec{S}) \cdot \vec{H} | \psi_g \rangle}{E_e^{(0)} - E_g^{(0)}}$$

$$+ \lambda^2 \frac{\langle \Psi_g | \vec{L} \cdot \vec{S} | \psi_e \rangle \langle \psi_e | \vec{L} \cdot \vec{S} | \psi_g \rangle}{E_e^{(0)} - E_g^{(0)}}$$

$$E^{(2)} = \frac{\langle \psi_g | \mu_B (\vec{L}+2\vec{S})\cdot\vec{H} + \lambda \vec{L}\cdot\vec{S} | \psi_e \rangle \langle \psi_e | \mu_B (\vec{L}+2\vec{S})\cdot\vec{H} + \lambda \vec{L}\cdot\vec{S} | \psi_g \rangle}{E_e^{(0)} - E_g^{(0)}}$$

$$\psi_g = \Phi_g(\vec{r}) \cdot \chi_s$$

$$\mathcal{H}^{(2)} = \sum_{ij} \Lambda_{ij} S_i S_j + \sum_i \underline{\Lambda'_{ij} H_i H_j S_i S_j}$$

Λ_{ij} are the generalized g factors.

$$g_{\text{matrix}} = \begin{pmatrix} g_{xx} & g_{xy} & g_{xz} \\ g_{yx} & g_{yy} & g_{yz} \\ g_{zx} & g_{zy} & g_{zz} \end{pmatrix} \longrightarrow \begin{pmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_3 \end{pmatrix}$$

$$g_1 = g_2 \neq g_3 \longrightarrow g_{\parallel} \ \& \ g_{\perp}$$

C. J. Ballhausen "Ligand Field Theory"

Susceptibility of the single ions (noninteracting) in a magnetic field.

single electron in a magnetic field

$E \uparrow$

$+g\mu_B H s$ $\longrightarrow$ $+\mu_B H$

$-g\mu_B H s$ $\qquad\qquad$ $-\mu_B H$

$$Z = e^{-\mu_B H\beta} + e^{+\mu_B H\beta} \qquad \beta = 1/k_B T$$

$$\langle M \rangle = kT \frac{d\ln Z}{dH} \qquad \langle m \rangle = \frac{-e^{-\mu\beta H} + e^{\mu\beta H}}{e^{-\mu\beta H} + e^{\mu\beta H}}$$

$$\langle M \rangle = \mu_B \frac{e^{\mu\beta H} - e^{-\mu\beta H}}{e^{\mu\beta H} + e^{-\mu\beta H}} = \mu_B \tanh(\mu\beta H)$$

$$\langle M \rangle = \mu_B \tanh(\mu \beta H)$$

Field strength that can be generated are generally a few Tesla

$$\mu \beta H = \mu H / kT \ll 1$$

and hence one can do a Taylor series expansion.

$$\tanh x \longrightarrow x \quad \text{in the limit } x \to 0$$

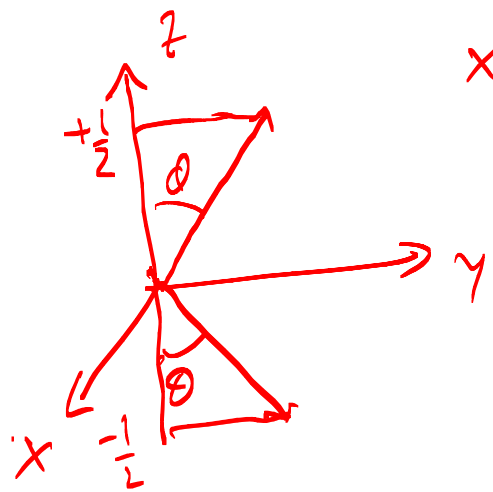
$$M = \frac{\mu_B^2 H}{kT} \quad \text{or} \quad \chi = \frac{\mu_B^2}{kT}$$

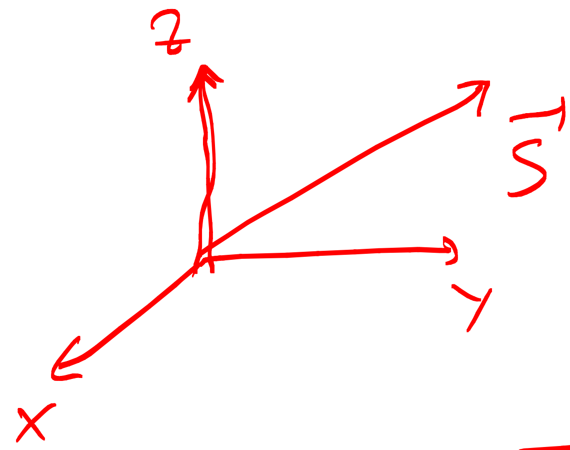
$$\chi \propto \frac{1}{T}$$

Let spin be very large

$[S_x, S_y] = i\hbar S_z$

and cyclic





$|\vec{S}|$, and one direction

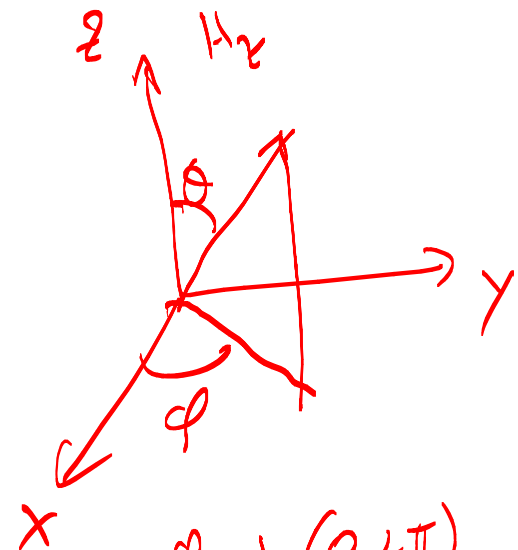
$$S = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)} > \frac{1}{2}$$

Torque $= \vec{M} \times \vec{H} \neq 0$

$$E = \mu_B \vec{S} \cdot \vec{H} = \mu H \cos\theta$$

$$\mu = g \mu_B S$$

$$Z = \int_0^{\pi} e^{\mu H \cos\theta \, \beta} \sin\theta \, d\theta$$



$\varphi \to (0, 2\pi)$

$\theta \to (0, \pi)$

$$\mu H \beta = \alpha$$

$$Z = \int_0^{\pi} e^{\alpha \cos\theta} \sin\theta \, d\theta$$

$y = \cos\theta \qquad dy = -\sin\theta \, d\theta$

$\theta = 0 \quad y = 1$

$\theta = \pi \quad y = -1$

$$Z = -\int_{1}^{-1} e^{\alpha y}\, dy$$

$$Z = \int_{-1}^{+1} e^{\alpha y}\, dy = \frac{1}{\alpha} e^{\alpha y}\Big|_{-1}^{1} = \frac{1}{\alpha}\left(e^{\alpha} - e^{-\alpha}\right).$$

$$M = kT \frac{d \ln Z}{dH}$$

H is the magnetic field

$$M = \mu . kT \frac{d \ln Z}{d\alpha} \cdot \frac{d\alpha}{dH}$$

$$M = \mu\left[\coth\left(\frac{\mu H}{kT}\right) - \frac{kT}{\mu H}\right]$$

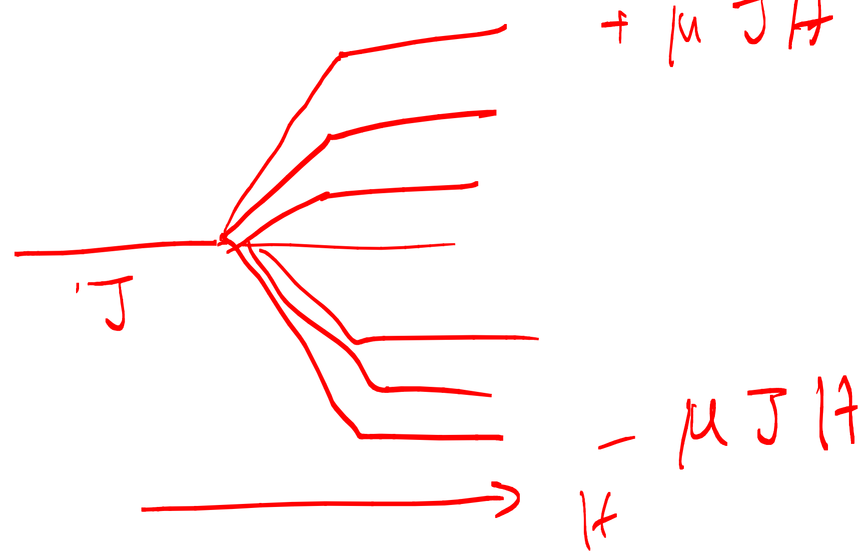
Langevin function $\quad L(x) = \left(\coth(x) - \frac{1}{x}\right)$

Limit $x \to 0 \quad L(x) = \left(\frac{x}{3} + \frac{1}{x} - \frac{1}{x}\right) = \frac{x}{3}$

$$M = \frac{\mu^2 H}{3kT} \qquad \chi = \frac{M}{H} = \frac{\mu^2}{3kT} \longrightarrow \text{Curie Law}$$

$$\chi \propto \frac{C}{T}$$

What happens when J is large but not necessarily very large



$$\mu = g\,\mu_B$$

Partition function:

$$Z = \underbrace{e^{-\mu J H \beta} + e^{-\mu (J-1) H \beta} + \cdots + e^{\mu J H \beta}}_{\text{geometric series}}$$

$$Z = e^{-\mu J H\beta}\left(1 + \cancel{e^{-\mu H\beta}} + \cancel{e^{-2\mu H\beta}} + \cdots + \cancel{e^{2\mu J H\beta}}\right)$$

$$e^{-\mu H\beta} Z = e^{-\mu J H\beta}\left(\cancel{e^{-\mu H\beta}} + \cancel{e^{-2\mu H\beta}} + \cdots + e^{(2\mu J H\beta - \mu H\beta)}\right)$$

$$Z\left(1 - e^{-\mu H\beta}\right) = e^{-\mu J H\beta}\left(1 - e^{-\mu(J+1)H\beta}\right)$$

$$= \frac{\sinh\,(2J+1)\mu H\beta/2}{\sinh(\mu H\beta/2)}$$

$x = J\mu H\beta$

$$M = kT\,\frac{d\ln Z}{dH} = kT\,\frac{d\ln Z}{dx}\cdot\frac{dx}{dH}$$

$$M = \mu\cdot(\mu J\beta)\;B_J(xJ)$$

where $B_J(y)$ is called Brillouin functi

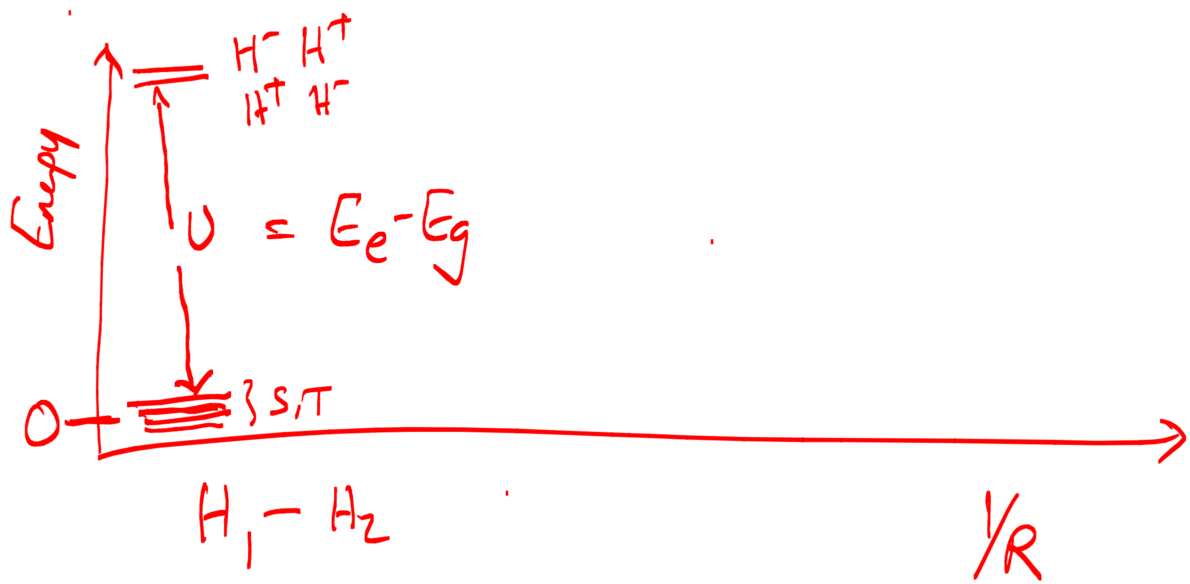
$$B_J(y) = \left[\frac{2J+1}{2J}\,\mathrm{Coth}\left(\frac{2J+1}{2J}\right)y - \frac{1}{2J}\,\mathrm{Coth}\,\frac{x}{2J}\right]$$

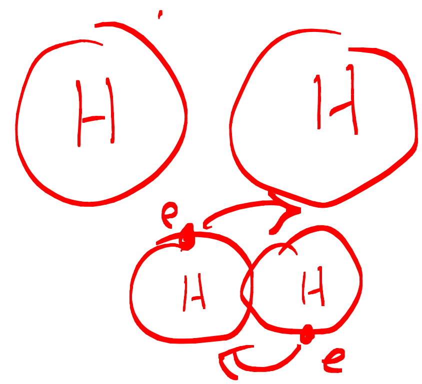
$$\chi = \frac{C}{T} = \frac{\mu_B^2 g^2 J(J+1)}{3kT}$$

$$\mu = g\mu_B\sqrt{J(J+1)}$$

$$= \mu^2/3kT$$

Cooperative Phenomena and Magnetism in metals and insulators





$$\langle HH | \hat{H} | H^+ H^- \rangle = t\sqrt{2}$$

$\langle \text{Singlet} \mid \text{Triplet} \rangle = 0$

$|S\rangle = \uparrow\downarrow - \downarrow\uparrow$

$|T\rangle = \uparrow\uparrow, \downarrow\downarrow, \uparrow\downarrow + \downarrow\uparrow$

$\mathcal{H}$ does not contain spin

Perturbation $\hat{\mathcal{H}}$ is a hopping of electron between atom

$$E^{(1)} = \langle \psi_g | \mathcal{H} | \psi_s \rangle = t \underbrace{\langle H-H | H^+ H^- \rangle}_{=0}$$

$$\mathcal{H} | H-H \rangle = |H^+ - H^-\rangle \quad |H^- - H^+\rangle$$

$$E^{(2)} = \sum_e \frac{\langle \psi_g | \mathcal{H} | \psi_e \rangle \langle \psi_e | \mathcal{H} | \psi_g \rangle}{-E_e + E_g}$$

$$= \frac{\langle H-H \,|\, \mathcal{H} \,|\, H^+H^- \rangle \langle H^+H^- \,|\, \mathcal{H} \,|\, H\text{-}H \rangle}{-U} + \frac{\langle H-H \,|\, \mathcal{H} \,|\, H^-H^+ \rangle \langle H^-H^+ \,|\, \mathcal{H} \,|\, H-H \rangle}{-U}$$

$$= \frac{2t^2}{U} + \frac{2t^2}{U} = \frac{4t^2}{U}$$

The energy of the singlet state is reduced by $\frac{4t^2}{U}$

But the triplet cannot mix with the excited states.

Hence nothing happens to them

$4t^2/U$ Triplet $S=1$

Singlet $S=0$

$$\Delta E_{ST} = \frac{4t^2}{U}$$

Using an effective Hamiltonian

$$S_1 \cdot S_2$$

$$S_{tot} = S_1 + S_2 = 0, 1$$

$$S_{tot}^2 = S_1^2 + S_2^2 + 2 S_1 \cdot S_2$$

or $S_1 \cdot S_2 = (S_{tot}^2 - S_1^2 - S_2^2)/2$

$S_1 = 1/2$ and $S_2 = 1/2$

$S_1 \cdot S_2$ for Triplet — has the eigenvalue

Triplet $S_{tot} = 1$

$$= \frac{1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right)}{2}$$

$$= \frac{2 - 3/2}{2} = 1/4$$

$S_{tot} = 0$

$$= \frac{(0)(0+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right)}{2}$$

$$= -3/4$$

$$\left(S_1 \cdot S_2 - 1/4\right)|S\rangle = -1$$

$$\left(S_1 \cdot S_2 - 1/4\right)|T\rangle = 0$$

$$\mathcal{H} = \frac{4t^2}{U}\left(S_1 \cdot S_2 - \tfrac{1}{4}\right) = J\left(S_1 \cdot S_2 - \tfrac{1}{4}\right)$$

$$E_T = 0 \qquad E_S = -\frac{4t^2}{U}$$

Kinetic Exchange $\longrightarrow$ P.W. Anderson

Ferromagnetic Exchange } two degenerate orbitals ϕ_1 and ϕ_2

Electrons are Fermions & hence the wave function of the many electron system must be antisymmetric

$$\Psi(1,2) = \underbrace{\Phi(r_1, r_2)}_{\substack{\text{Symmetric} \\ \text{anti-symmetric}}} \; \underbrace{\chi(s_1, s_2)}_{\substack{\text{anti-symmetric} \\ \text{symmetric}}}$$

For triplets
spatial part is antisymmetric & spin part is symmetric

$$\Phi_T(r_1, r_2) = \left(\phi_1(r_1)\,\phi_2(r_2) - \phi_1(r_2)\,\phi_2(r_1)\right)$$

$$\chi_T(s_1, s_2) = \alpha_1\,\alpha_2$$

$$\Psi_T = \Phi_T(r_1, r_2)\;\chi_T(s_1, s_2)$$

$$\Psi_S = \Phi_S(r_1, r_2)\ \chi_S(s_1, s_2)$$

$$= \left(\phi_1(r_1)\phi_2(r_2) + \phi_1(r_2)\,\phi_2(r_1)\right)(\alpha_1\beta_2 - \beta_1\alpha_2)$$

Perturbation is $\dfrac{e^2}{r_{12}}$; Although unperturbed Singlets & Triplets are degenerate they do not mix & we can carry out non-degenerate P.T.

$$E_T = \left\langle \Psi_T \left| \frac{e^2}{r_{12}} \right| \Psi_T \right\rangle \quad ; \quad E_S = \left\langle \Psi_S \left| \frac{e^2}{r_{12}} \right| \Psi_S \right\rangle$$

$$A = \int \phi_1^*(1)\,\phi_1(1)\,\frac{e^2}{r_{12}}\,\phi_2^*(2)\,\phi_2(2)\,d^3r_1\,d^3r_2\,ds_1\,ds_2$$

Coulomb integral

$$= \int \rho_1(1)\,\rho_2(1)/r_{12}\;d^3r_1\,d^3r_2$$

$$J = \int \phi_1^*(1)\,\phi_2(1)\,\frac{e^2}{r_{12}}\,\phi_2^*(2)\,\phi_1(2)\,d^3r_1\,d^3r_2\,ds_1\,ds_2$$

Exchange integral

$J \to 0$ for paired spins because we have opposite spins for electrons 1 and 2

$\int \alpha_1 \beta_2\, ds_1 = 0$ $\int \alpha_2 \beta_1\, ds_2 = 0$

$E_S = A$

$E_T = A - 2J$

A and J are both positive

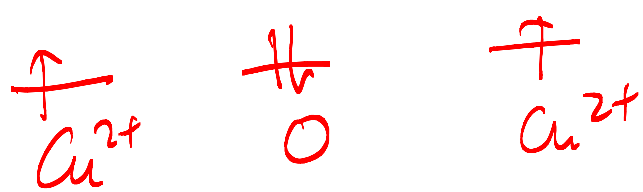
$\therefore$ E_T has lower energy than E_S

$\longrightarrow$ Hund's rule

Magnetism is a consequence of direct vs kinetic exchange

$$\mathcal{H}_{eff} = A - 2J(s_1 \cdot s_2 - 1/4) \longrightarrow \text{Dirac - van Vleck - Heisenberg Exchange}$$

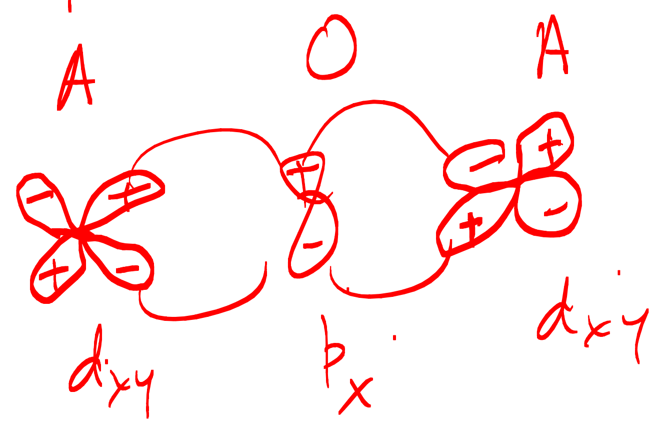
LL' Q-M



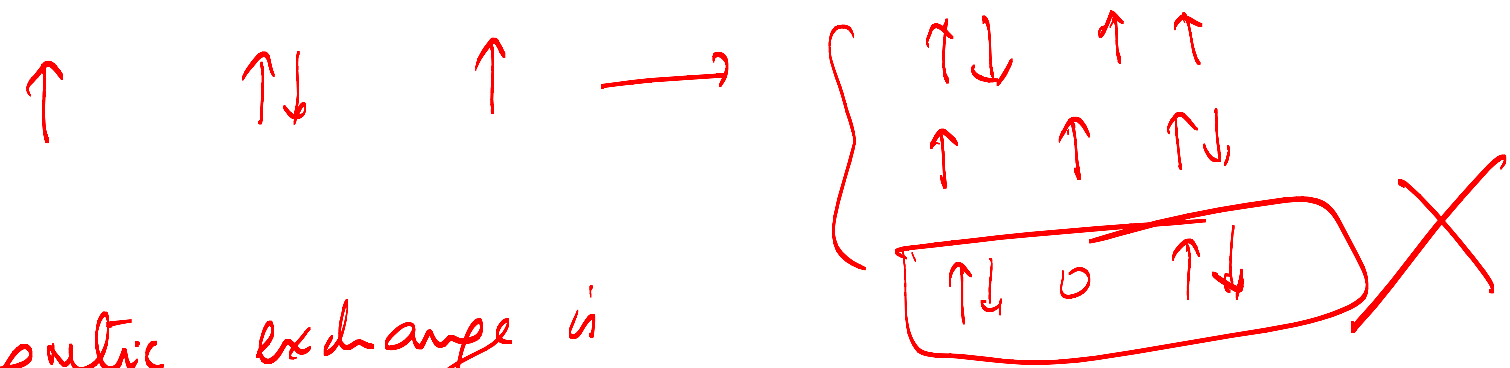
Triplet state

Singlet state

$\uparrow \qquad \uparrow\downarrow \qquad \downarrow$

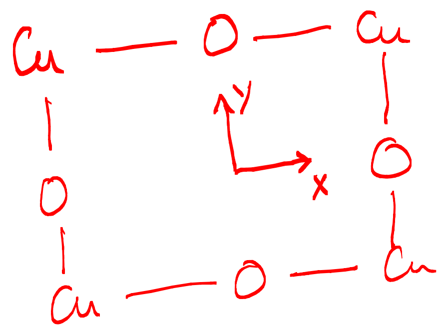


$$S=0 \qquad \uparrow \quad \uparrow\downarrow \quad \downarrow \longrightarrow \left\{ \begin{array}{ccc} \uparrow\downarrow & \uparrow & \downarrow \\ \uparrow & \downarrow & \uparrow\downarrow \\ \uparrow\downarrow & 0 & \uparrow\downarrow \end{array} \right.$$

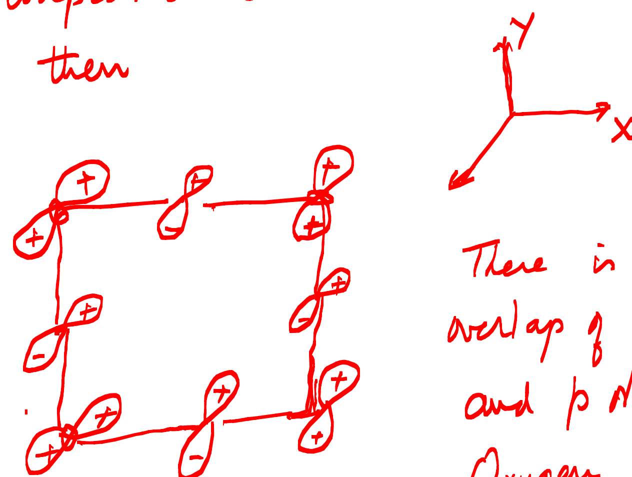


Antiferromagnetic exchange is favored

Cu^{2+} is with J-T distortion and the orbital in which the unpaired electron is residing in d_{z^2} and Cu-O-Cu angle is 90° as in CuO_2 plane

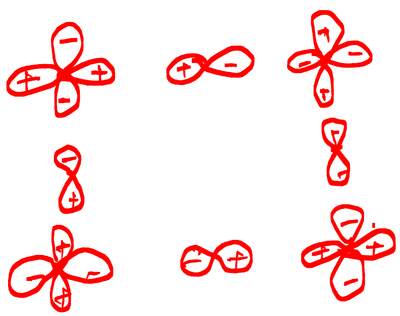


1. If the unpaired electron is in the d_{z^2} Orbital, then

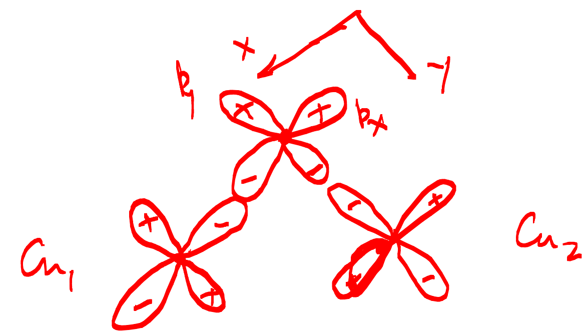


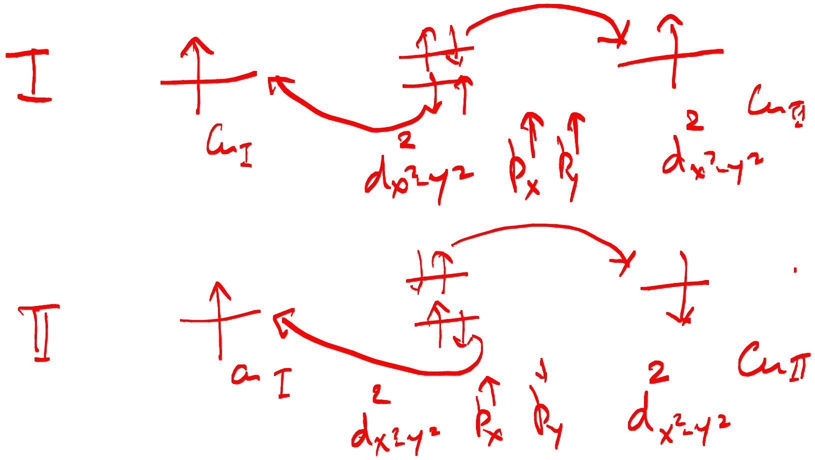
There is no overlap of the d_{z^2} and p orbitals on Oxygen and the system will be weakly ferromagnetic

2. If the unpaired electron is in the $d_{x^2-y^2}$ orbital then



The overlap of the $d_{x^2-y^2}$ orbitals with the p_x orbitals of Oxygen in the x-direction and p_y orbitals of oxygen in y-direction leads to strong antiferromagnetic interaction.

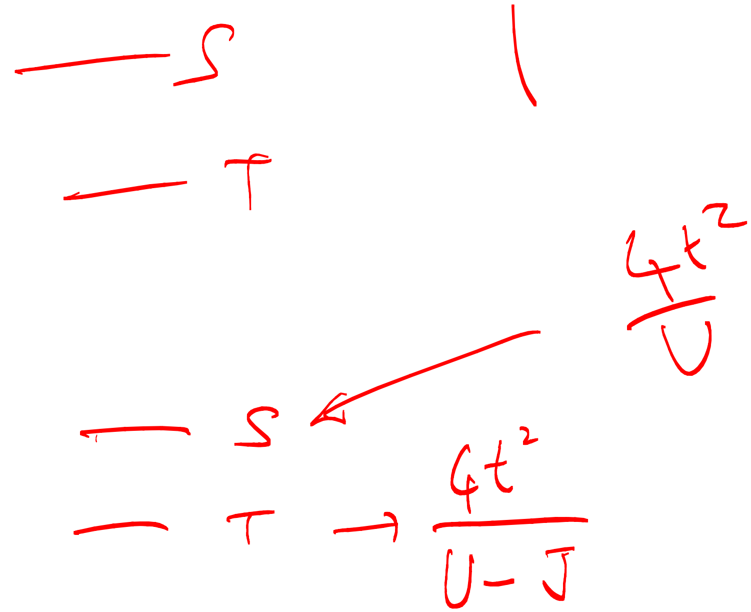




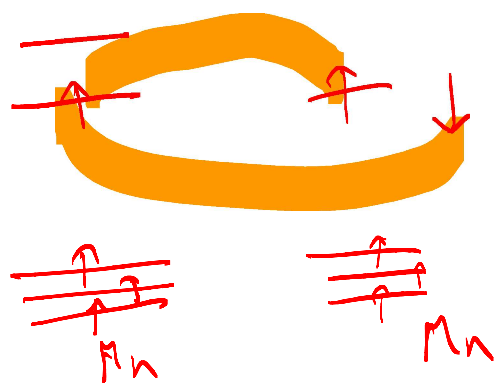
If the Cu–O–Cu is 90°, then the $d_{x^2-y^2}$ orbital of Cu_I overlaps with the p_x orbital of oxygen while the $d_{x^2-y^2}$ orbital of Cu_{II} overlaps with the p_y orbital of the oxygen. Then the virtual state generated if the two Cu electrons are parallel then it is I. If they are antiparallel then the configuration generated is II. By Hund's rule the intermediate state I is more stable than the intermediate state II.

Ferromagnetic exchange wins even though overlap is nonzero as it is with different p-orbitals.

McConnell



Double exchange interaction

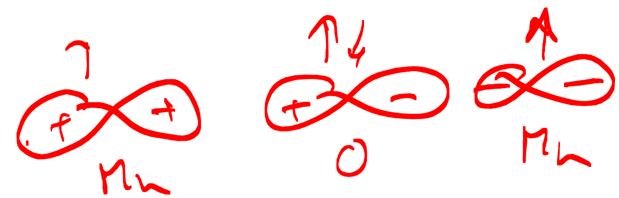


Mn^{3+}

Mn - O - Mn

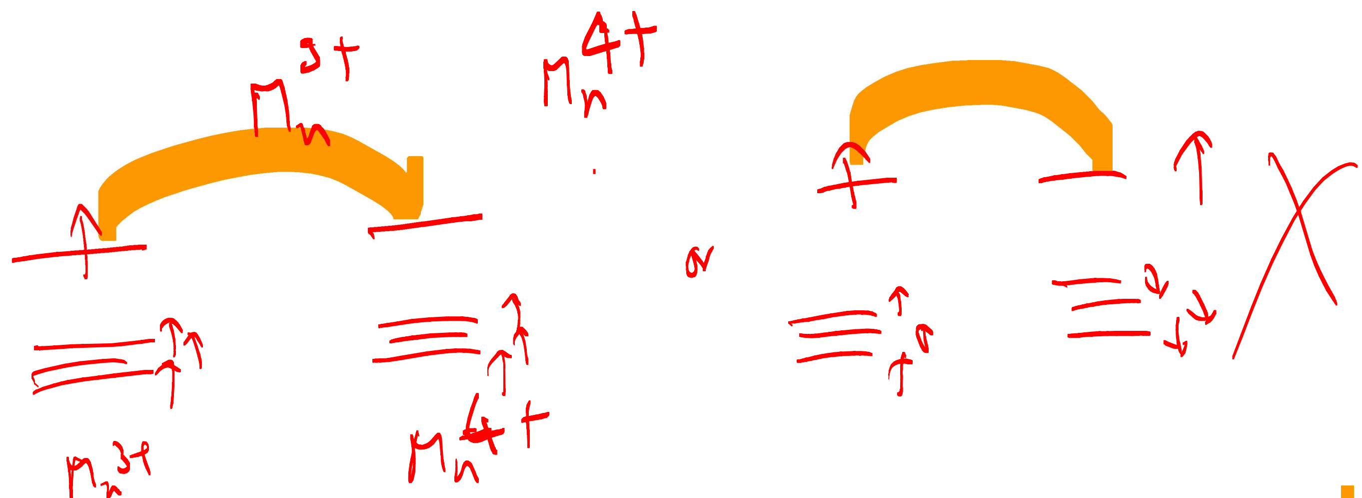
$LaMnO_3$

Mn–O–Mn in antiferromagnetic

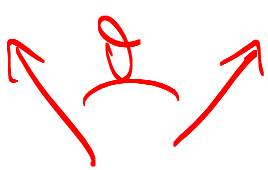


Mn - Mn interaction

$La_{1-x}Sr_xMnO_3$



$t \rightarrow t_0 \cos\theta$



Ferromagnetic system

Double exchange

$$\mathcal{H} = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j$$

Let us consider the interaction is ferromagnetic

$$\sum_i J \vec{S}_i \cdot \underline{\langle \vec{S}_j \rangle}$$

$\langle \vec{S}_j \rangle$ is an average or expectation value of the spin at site j

$$\mathcal{H}_0 + \mathcal{H}_a = \sum_i (\mathcal{H}_a + J\langle S \rangle) \cdot S_i$$

$$\langle S_i \rangle = \frac{1}{N} \sum_j \langle S_j \rangle$$

The internal field $H_{int} = \lambda M \qquad M = N\langle S \rangle$

The total field experienced by a spin is

$$\left(H_a + H_{int}\right) = \left(H_a + \lambda M\right)$$

$$\frac{C}{T} = \frac{M}{(H_a + \lambda M)}$$

$$C H_a + C\lambda M - MT = 0$$

$$M(C\lambda - T) = -C H_a$$

experimental susceptibility $\Longrightarrow$ $\dfrac{M}{H_a} = \dfrac{C}{T - C\lambda}$

$$\chi = \frac{C}{T - C\lambda} \qquad C\lambda = \theta$$

$$\chi = \frac{C}{T-\theta}$$ is the expression for susceptibility of a ferromagnet

$$\mathcal{H} = \sum_{\langle i,j \rangle} J \, S_i^A \cdot S_j^B$$

J is +ve for a/m

i is on sublattice A
and j is on sublattice B

S_i^A is in a field of spins on B sublattice

S_j^B are in a field of spins on A sublattice.

$$H_A^{tot} = (H_a - \mu M_B)$$

$$H_B^{tot} = (H_a - \mu M_A)$$

the internal field acts against H_a in both the sublattices

$$\frac{M_A}{H_A} = \frac{C_A}{T} \; ; \qquad \frac{M_A}{H_B} = \frac{C_B}{T}$$

$$M_A T = (H_a - \mu M_B)\, C_A$$

$$M_B T = (H_a - \mu M_A)\, C_B$$

$$[M_A T + \mu M_B C_A = H_a C_A] \times \mu C_B$$

$$[M_B T + \mu M_A C_B = H_a C_B]\; T$$

$$M_A T \mu C_B + \mu^2 M_B C_A C_B = H_a \mu C_A C_B$$

$$M_B T^2 + \mu M_A C_B T = H_a C_B T$$

$$M_B\left(-T^2+\mu^2 C_A C_B\right) = H_a C_B\left(\mu C_A - T\right)$$

$$M_B = \frac{H_a C_B(\mu C_A - T)}{(\mu^2 C_A C_B - T^2)}$$

$$M_A = \frac{H_a C_A(\mu C_B - T)}{(\mu^2 C_A C_B - T^2)}$$

$$\langle M\rangle = M_A + M_B = \frac{H_a\left[\mu C_B C_A - T C_A + \mu C_A C_B - T C_B\right]}{(\mu^2 C_A C_B - T^2)}$$

$$\frac{\langle M \rangle}{H_a} = \chi = \left. \frac{[2\mu C_B C_A - T C_A - T C_B]}{[\mu^2 C_A C_B - T^2]} \right\} \text{ is for ferrimagnets}$$

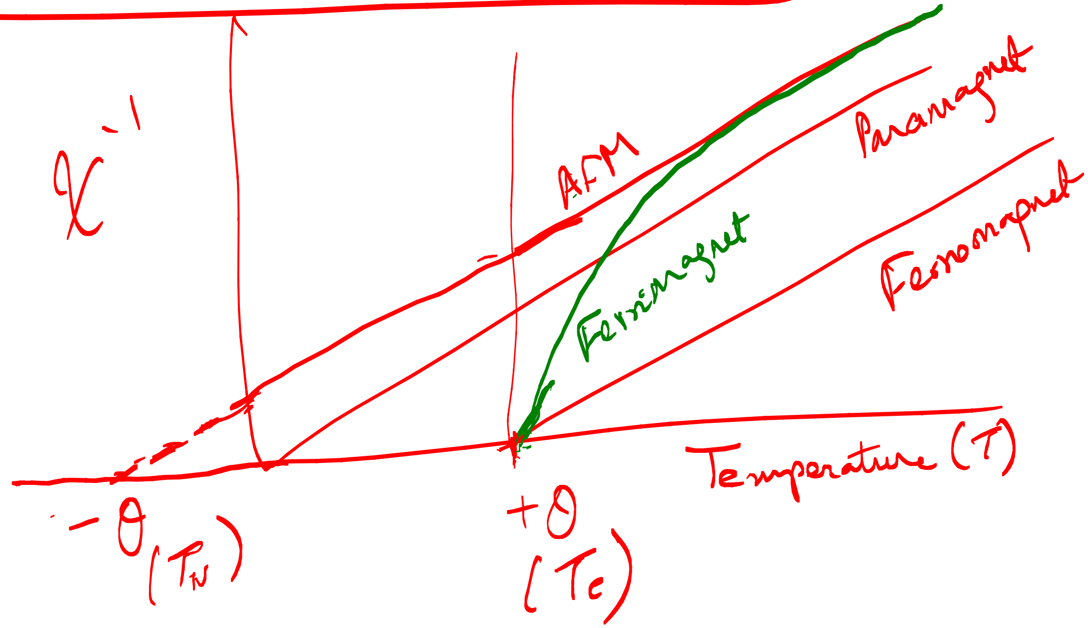
For antiferromagnets $C_A = C_B = C$

$$\chi_{afm} = \frac{2[\mu C^2 - TC]}{[\mu^2 C^2 - T^2]}$$

$$= \frac{2\mu C \cancel{[\mu C - T]}}{[\mu C + T]\cancel{[\mu C - T]}} = \frac{2\mu C}{(\mu C + T)}$$

$$\boxed{\chi_{afm} = \frac{C}{T+\theta}}$$

$\theta = \mu C$



$T < +\theta$

χ^{-1} is not defined as it is $\rightarrow \infty$