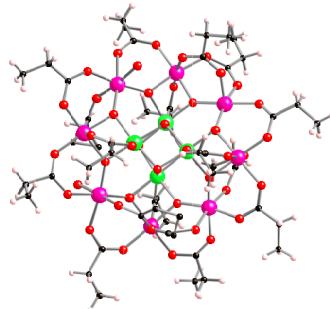


# Exchange, Super-Exchange Interaction and Magnetic Anisotropy: a Theoretical Insight



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Bangalore : 26-30 November 2018

# A POSSIBLE ROLE OF QUANTUM CHEMISTRY IN MAGNETISM

$$\hat{H}\Psi = E\Psi$$

Quantum chemists work with the exact electronic Hamiltonian (and add relativistic effects):

$$\hat{H}_{BO} = \sum_{A < B} \frac{Z_A Z_B}{R_{AB}} - \sum_i \frac{A_i}{2} - \sum_{A,i} \frac{Z_A}{r_{iA}} + \sum_{i < j} \frac{1}{r_{ij}} = cste + \sum_i \vartheta_i + \sum_{i < j} \frac{1}{r_{ij}}$$

Responsible for  
the difficulty

The methodological corpus developed to solve the Schrödinger equation is particularly important.  
It consists in treating the electron repulsion as best as possible, i.e. to account for **electron correlation**.

However:

- i) it cannot be applied to large magnetic systems,
- ii) the interpretation of the wavefunctions is often complicated.
- iii) There are by far too many interactions and configurations to model simply the properties...

In practice, both experimentalists and theoretical physicists (who add collective effects in solids) use model Hamiltonians.

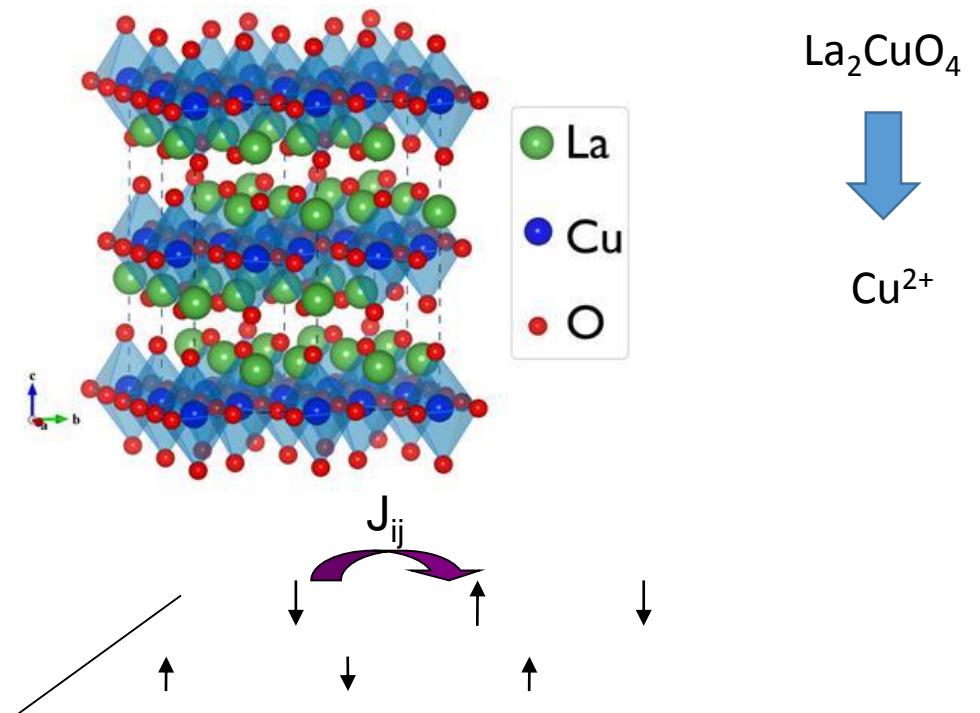
# The Heisenberg Hamiltonian is a model Hamiltonian

## Why do we need models?

Model Hamiltonians are simpler than the exact electronic Hamiltonian:

$$\text{Ex: } \hat{H} = - \sum_{\langle i,j \rangle} J_{ij}^{\text{eff}} \hat{S}_i \cdot \hat{S}_j$$

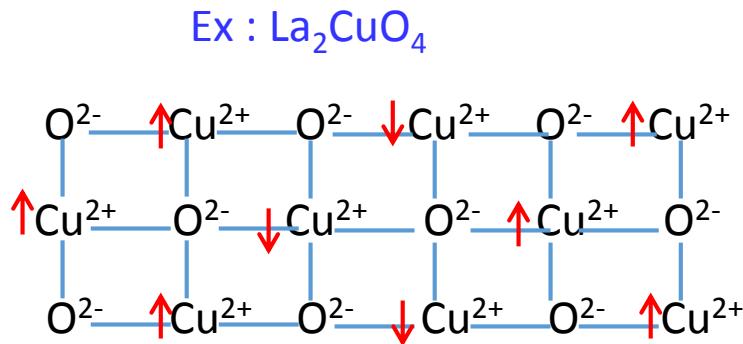
- They only treat a few number of **electrons** (here 1 per copper ion)
- They only keep a few number of **configurations** (here spin distributions)
- They have **effective interactions** that are supposed to capture all the neglected physics



Objective : **derive analytically** and determine from ***ab initio* calculation** and the **effective Hamiltonian theory** the physical content of the exchange interaction and the Zero Field Splitting (magnetic anisotropy).

# Magnetic systems and the Heisenberg Hamiltonian

Low-energy spectrum : states which only differ by their spin and have the same spatial configuration.



All orbitals are doubly occupied except  $d_{x^2-y^2}$  of Cu<sup>2+</sup> that carries a single electron.  
From the different spin distributions that can be build by distributing all  $\uparrow$  and  $\downarrow$  on the skeleton, one can generate all the lowest states.

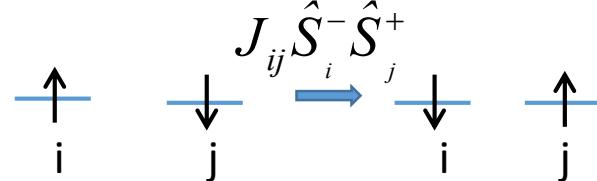
These states are generally quasi-degenerate

These states are well described by the Heisenberg Hamiltonian Dirac van Vleck (HDvV) model

$$\hat{H} = -2 \sum_{i < j} J_{ij} \left( \hat{S}_i \hat{S}_j - \frac{\hat{I}}{4} \right)$$

with  $\hat{S}_i \hat{S}_j = \hat{S}_{zi} \hat{S}_{zj} + \frac{I}{2} [\hat{S}_i^+ \hat{S}_j^- + \hat{S}_i^- \hat{S}_j^+]$

The exchange integral of  $J_{ij} \neq 0$  if i and j interact :



# Heisenberg model

The model space is constituted of all neutral determinants, products of local ground states, i.e. local highest spin states (due to the Hund's rule).

$$\hat{H}^{Heis.} = -\sum_{\langle ij \rangle} J_{ij} \left( \hat{\vec{S}}_i \cdot \hat{\vec{S}}_j - \frac{\hat{n}_i \hat{n}_j}{4} \right)$$

This definition ensures that the highest spin state is at zero of energy here  $\hat{n}_i$  is the number of unpaired electron(s) on site i

$$\begin{aligned} \hat{\vec{S}}_i \cdot \hat{\vec{S}}_j &= \begin{pmatrix} \hat{S}_{X_i} & \hat{S}_{Y_i} & \hat{S}_{Z_i} \end{pmatrix} \begin{pmatrix} \hat{S}_{X_j} \\ \hat{S}_{Y_j} \\ \hat{S}_{Z_j} \end{pmatrix} \\ &= \hat{S}_{X_i} \hat{S}_{X_j} + \hat{S}_{Y_i} \hat{S}_{Y_j} + \hat{S}_{Z_i} \hat{S}_{Z_j} = \hat{S}_{Z_i} \hat{S}_{Z_j} + \frac{I}{2} (\hat{S}_i^+ \hat{S}_j^- + \hat{S}_i^- \hat{S}_j^+) \end{aligned}$$

Heisenberg Hamiltonian matrix for 2 electrons in 2 orbitals : Ex:  $S_1=1/2$ ,  $S_2=1/2$

$$\hat{H}^{\text{Heis.}} = -J_{12} \left( \hat{S}_{z1} \hat{S}_{z2} - \frac{\hat{I}}{4} \right) - \frac{J_{12}}{2} \left( \hat{S}_{+1} \hat{S}_{-2} + \hat{S}_{-1} \hat{S}_{+2} \right)$$

$$H_{2l} = \langle \downarrow_1 \uparrow_2 | \hat{H} | \uparrow_1 \downarrow_2 \rangle$$

Space:

- $Ms=1$  distribution :  $\uparrow\uparrow$ ,  $Ms=1$  component of the triplet
- $Ms=0$  2 possible distributions :  $\downarrow\uparrow$ ,  $\downarrow\uparrow$ ,  $Ms=0$  component of the triplet and one singlet state.
- $Ms=-1$  distribution :  $\downarrow\downarrow$ ,  $Ms=1$  component of the triplet

Diagonal elements :

$$\langle \uparrow_1 \uparrow_2 | -J \left( \hat{S}_{z1} \hat{S}_{z2} - \frac{1}{4} \right) | \uparrow_1 \uparrow_2 \rangle = -J \left( \frac{1}{4} - \frac{1}{4} \right) \langle \uparrow\uparrow \| \uparrow\uparrow \rangle = 0$$

$$\langle \uparrow\downarrow | -J \left( \hat{S}_{z1} \hat{S}_{z2} - \frac{1}{4} \right) | \uparrow\downarrow \rangle = -J \left( -\frac{1}{4} - \frac{1}{4} \right) \langle \uparrow\downarrow \| \uparrow\downarrow \rangle = \frac{J}{2}$$

Off-diagonal elements :

$$\langle \downarrow\uparrow | -\frac{J}{2} \left( \hat{S}_1^+ \hat{S}_2^- + \hat{S}_1^- \hat{S}_2^+ \right) | \uparrow\downarrow \rangle = -\frac{J}{2} \sqrt{s_1(s_1+1)-m_{s1}(m_{s1}-1)} \sqrt{s_2(s_2+1)-m_{s2}(m_{s2}+1)} \langle \downarrow\uparrow \| \downarrow\uparrow \rangle = -\frac{J}{2} \langle \downarrow\uparrow \| \downarrow\uparrow \rangle$$

The matrix writes :

$$(\hat{H}^{\text{Heis.}}) = \begin{pmatrix} \langle \uparrow\downarrow | & | \uparrow\downarrow \rangle & | \downarrow\uparrow \rangle & | \uparrow\uparrow \rangle & | \downarrow\downarrow \rangle \\ \langle \downarrow\uparrow | & J/2 & -J/2 & 0 & 0 \\ \langle \uparrow\uparrow | & -J/2 & J/2 & 0 & 0 \\ \langle \downarrow\downarrow | & 0 & 0 & 0 & 0 \end{pmatrix}$$

# Eigenvalues and eigenvectors

The  $M_s=-1$  and  $M_s=+1$  of the triplet are eigenvectors and their energy is zero

Eigenvalues in the  $M_s=0$  subspace ( $2 \times 2$  matrix) :

$$\begin{array}{c} |\uparrow\downarrow\rangle \quad |\downarrow\uparrow\rangle \\ \langle\uparrow\downarrow| \quad \langle\downarrow\uparrow| \end{array} \begin{pmatrix} J/2 & -J/2 \\ -J/2 & J/2 \end{pmatrix} \xrightarrow{\hspace{1cm}} \begin{vmatrix} J/2-E & -J/2 \\ -J/2 & J/2-E \end{vmatrix} = 0$$

Eigenvalues:  $(J/2-E)^2 - (J/2)^2 = 0 \Leftrightarrow E = \begin{cases} 0 & \text{Energy of the } M_s=0 \text{ component of the triplet.} \\ J & \text{Energy of the singlet} \end{cases} \quad \Delta E_{ST} = J$

$$\hat{H}\Psi = E\Psi \quad \begin{pmatrix} J/2 & -J/2 \\ -J/2 & J/2 \end{pmatrix} \begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} = E_S \begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} \Leftrightarrow \begin{cases} (J/2)C_{11} - (J/2)C_{12} = E_S C_{11} \\ |C_{11}|^2 + |C_{12}|^2 = 1 \end{cases} \Leftrightarrow \begin{cases} C_{11} = \pm C_{12} \\ C_{12} = \frac{1}{\sqrt{2}} \end{cases}$$

Eigenvectors:

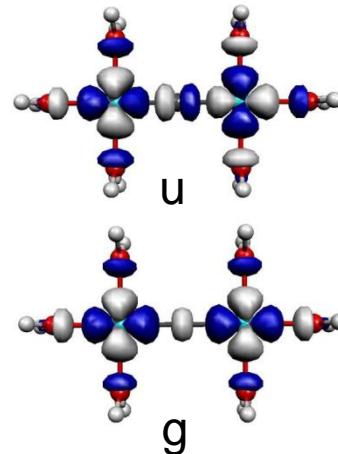
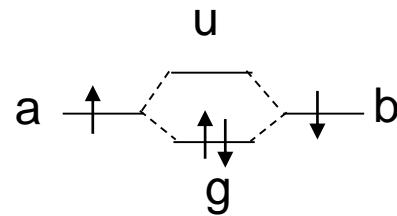
$$|\Psi_s\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$|\Psi_t\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

# Ab initio calculations : magnetic systems are correlated systems

Hückel description (monoelectronic) is not valid.

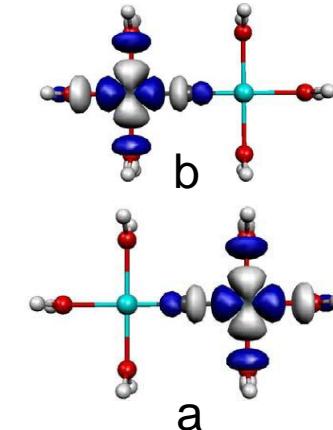
Delocalized orbitals



Magnetic orbitals

$$a = \frac{g+u}{\sqrt{2}} \quad b = \frac{g-u}{\sqrt{2}}$$

$$\langle a | b \rangle = 0$$



The single determinant function  $|g\bar{g}|$  is no more a good function

$$|g\bar{g}| = \frac{1}{2} |a\bar{a} + b\bar{b} + a\bar{b} + b\bar{a}|$$

Same amount of ionic and neutral configurations

$ a\bar{a} $	$\uparrow\downarrow$	$-\_$	Ionic
$ b\bar{b} $	$-\_$	$\uparrow\downarrow$	
$ a\bar{b} $	$\uparrow$	$\downarrow$	
$- b\bar{a} $	$\downarrow$	$\uparrow$	
$\text{Cu}^{3+}\text{Cu}^+$			Neutral
$\text{Cu}^{2+}\text{Cu}^{2+}$			

For strongly localized electrons : multireference wavefunctions are required to restore the correct ionic/neutral ratio

$$^1\psi_{CAS} = \lambda|g\bar{g}| - \mu|u\bar{u}| = \left(\frac{\lambda+\mu}{2}\right)|a\bar{b} - \bar{a}b| + \left(\frac{\lambda-\mu}{2}\right)|a\bar{a} - \bar{b}b|$$

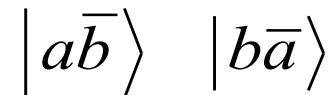
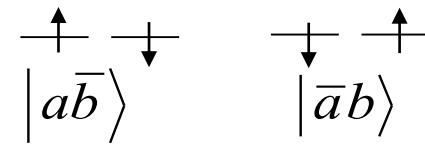
$$^3\psi_{CAS} = |gu| = |ab|$$

1%

Covalent (or ionic) bond:  $\lambda \gg \mu$   
Magnetic « bond » :  $\lambda \sim \mu$

## Derivation of the Heisenberg model from the exact electronic Hamiltonian

Heis. works on neutral



$$\begin{pmatrix} \langle ab | & \end{pmatrix} \begin{bmatrix} J_{ab} & K \\ K & J_{ab} \end{bmatrix} \begin{pmatrix} | a'b \rangle & | b'a \rangle \end{pmatrix}$$

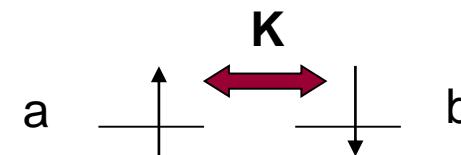
$$(J_{ab} - E)^2 - (K)^2 = 0 \Leftrightarrow E = \begin{cases} J_{ab} + K & \text{Singlet} \\ J_{ab} - K & \text{Triplet} \end{cases}$$

Repulsion of the two electrons in the orbitals a and b

$$J_{ab} = \iint a(1)b(2) \frac{1}{r_{12}} b(2)a(1) d\tau_1 d\tau_2$$

Exchange integral of the exact electronic Hamiltonian :

$$K = \iint a(1)b(2) \frac{1}{r_{12}} b(1)a(2) d\tau_1 d\tau_2 > 0$$



The triplet is lower in energy than the singlet : Hund's rule

$$\Delta E_{ST} = 2K$$

If one limits to the neutral determinants, the world would be ferromagnetic !

# What is the microscopic origin of antiferromagnetism ?

Kinetic exchange : ionic determinants contribution to the singlet state

	$ ab\rangle$	$ b\bar{a}\rangle$	$ a\bar{a}\rangle$	$ b\bar{b}\rangle$		$ S_N^g\rangle$	$ T_N^u\rangle$	$ S_I^s\rangle$	$ S_I^u\rangle$		
neutral	$\begin{cases} J_{ab} & K \\ K & J_{ab} \end{cases}$	$\begin{cases} t & t \\ t & t \end{cases}$				$\begin{cases} K & 0 & 2t & 0 \\ 0 & -K & 0 & 0 \\ 2t & 0 & U+K & 0 \\ 0 & 0 & 0 & U-K \end{cases}$					$U=J_{aa}-J_{ab}$
Ionic	$\begin{cases} t & t \\ t & t \end{cases}$	$\begin{cases} J_{aa} & K \\ K & J_{aa} \end{cases}$									$t=F_{ab}$ can be seen as the hopping integral (b of Hückel) (actually it is bielectronic)
	$\uparrow \downarrow$	$\uparrow \downarrow$									
	$\downarrow \uparrow$	$\downarrow \uparrow$									

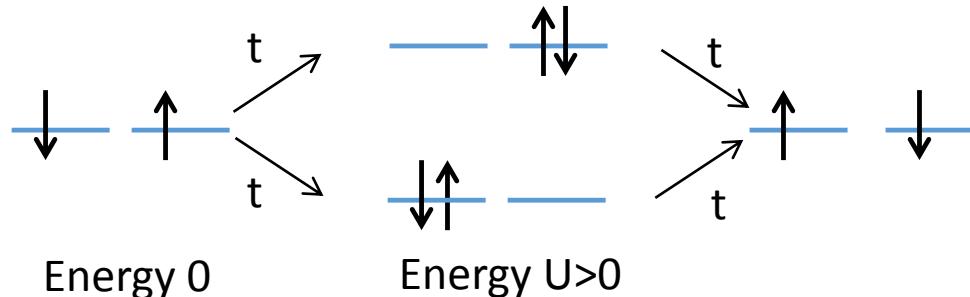
Variationally

$$E(S^g) = \frac{U + 2K - \sqrt{16t^2 + U^2}}{2}$$

$$E(T) = -K$$

Perturbatively (2<sup>nd</sup> order) :

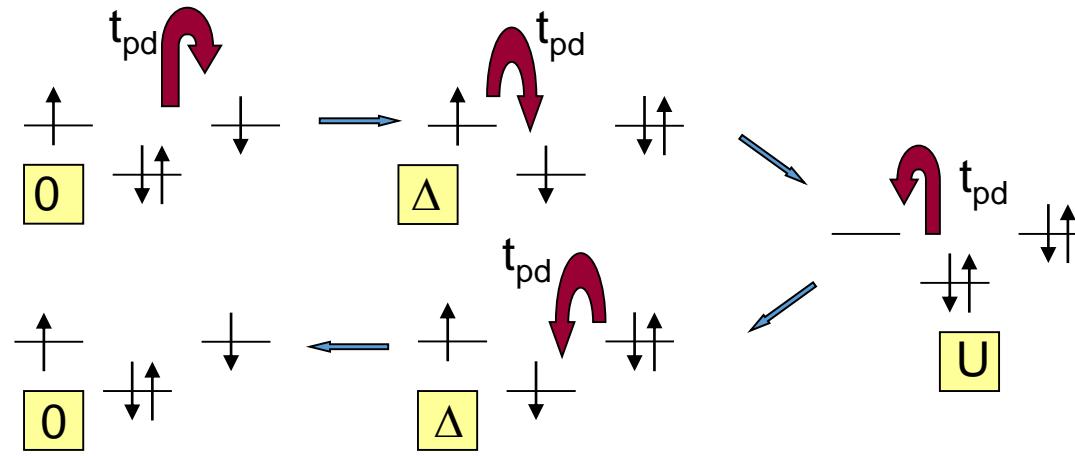
$$\left. \begin{aligned} E(S_N^g) &= K - \frac{4t^2}{U} \\ E(T_N^u) &= -K \end{aligned} \right\} \Delta E_{ST} = 2K - \frac{4t^2}{U} = J$$



Si  $K > 2t^2/U \rightarrow J>0 \rightarrow$  ferromagnetism  
Si  $K < 2t^2/U \rightarrow J<0 \rightarrow$  antiferromagnetism

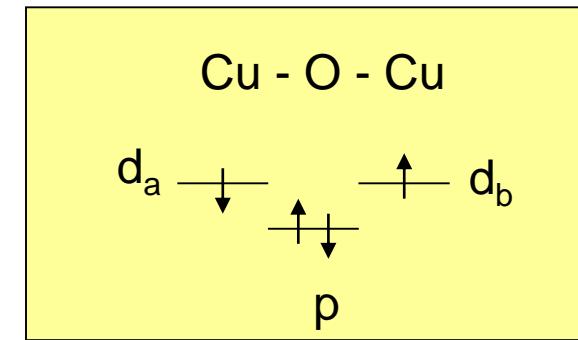
# Role of the bridging ligands : superexchange

Superexchange : through ligand coupling



$\Delta$  : charge transfer (MLCT or LMCT) det. energy

U : ionic det. energy



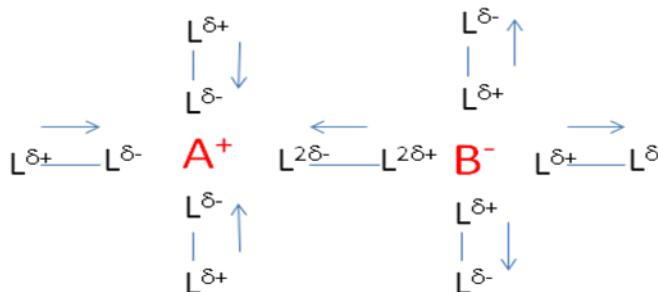
$$J = 2K - \frac{4}{U} \frac{t_{pd}^4}{\Delta^2} - 4 \frac{t_{ab}^2}{U}$$

The coupling may also involve antibonding orbitals, for instance  $\pi^*$  in molecular bridge

To correlate the ML- or LM-CT configurations is necessary to describe correctly their effect

## Other effects of electron correlation: charge and spin polarization

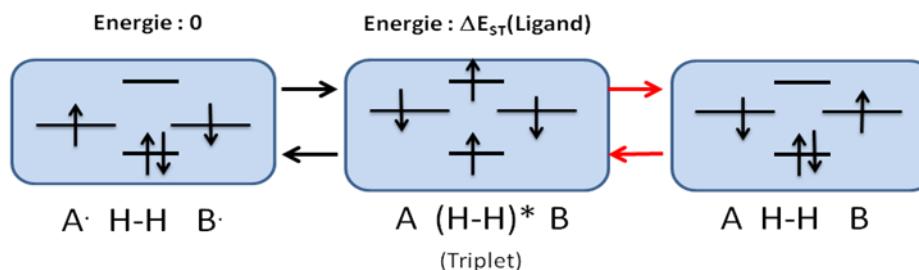
Charge polarization : The screening of U changes U by  $U^{\text{eff}}$



$$J = \Delta E_{ST}^{\text{2nd order}} = 2K - \frac{4t_{ab}^2}{U - \sum_{i,r} \frac{\langle i | J_a - J_b | r \rangle^2}{(E_r - E_i) + U}} = 2K - \frac{4t_{ab}^2}{U^{\text{eff}}}$$

The screening of U is an antiferromagnetic contribution, it also comes from higher excitations. It is huge : J may be multiplied by 3 to 10

Spin polarization : Changes K to an effective  $K^{\text{eff}} = K - \Delta K$



$$\begin{pmatrix} \left| \bar{i} \bar{i} a \bar{b} \right\rangle & \left| \bar{i} \bar{i} \bar{a} b \right\rangle \\ \left\langle \bar{i} \bar{i} a \bar{b} \right| & \left\langle \bar{i} \bar{i} \bar{a} b \right| \end{pmatrix} \begin{pmatrix} E_{PS} & -K_{ab} - \Delta K \\ -K_{ab} - \Delta K & E_{PS} \end{pmatrix}$$

$$\Delta K = 2 \sum_{i,r} \frac{\left\langle \bar{i} \bar{i} a \bar{b} \right| \hat{H} \left| \bar{i} r a \bar{b} \right\rangle \left\langle \bar{i} r a \bar{b} \right| \hat{H} \left| \bar{i} \bar{i} a \bar{b} \right\rangle}{\varepsilon_r - \varepsilon_i} = 2 \sum_{i,r} \frac{\langle i | K_a | r \rangle \langle r | K_b | i \rangle}{\varepsilon_r - \varepsilon_i}$$

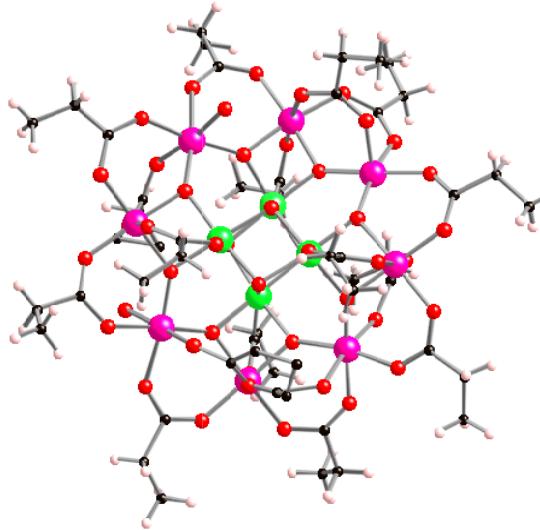
$\Delta K$  may have both signs and may therefore stabilize either the singlet or the triplet

# Conclusions on the exchange and super exchange interactions : $J^{\text{eff}}$ made of several contributions

- i) Direct exchange is always positive and favors ferromagnetic couplings
- ii) Kinetic exchange always favors antiferromagnetic couplings
- iii) The super-exchange goes through the bridging ligands
- iv) Charge polarisation favors antiferromagnetic couplings
- v) Spin polarisation might either be in favor of ferro or antiferromagnetic couplings
- vi) All these contributions are very sensitive to electron correlation

Magnetic Interactions in Molecules and Highly Correlated Materials: Physical Content, Analytical Derivation, and Rigorous Extraction of Magnetic Hamiltonians; [Jean Paul Malrieu](#), [Rosa Caballol](#), [Carmen J. Calzado](#), [Coen de Graaf](#), and [Nathalie Guihéry](#); *Chem. Rev.*, 2014, 114 (1), pp 429–492 and references here in.

# Magnetic anisotropy : Microscopic origin of single molecule magnet behaviour

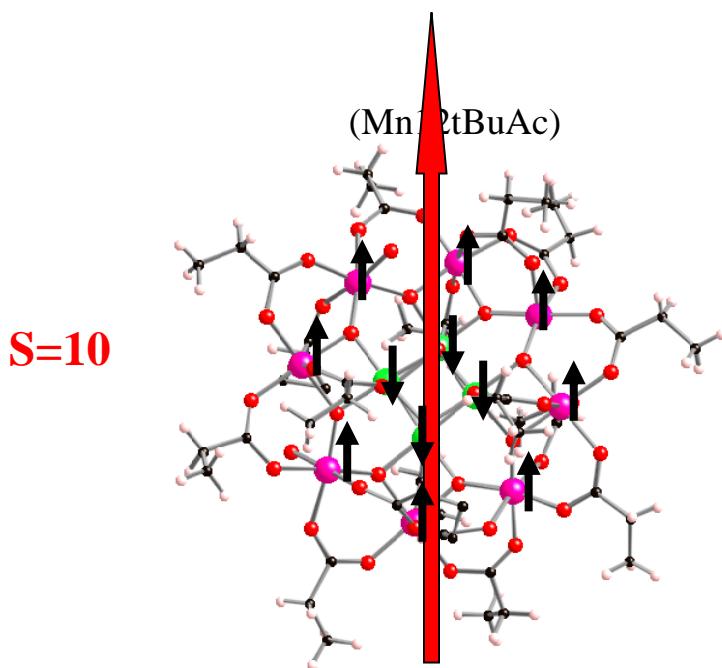


Methodology of quantum chemistry to get accurate ZFS parameters and method of extraction based on the effective Hamiltonian theory and *ab initio* results (now implemented in ORCA)

- R. Maurice, N. Guihéry, R. Bastardis and C. de Graaf J. Chem. Theory Comput., 6, 55 (2010)
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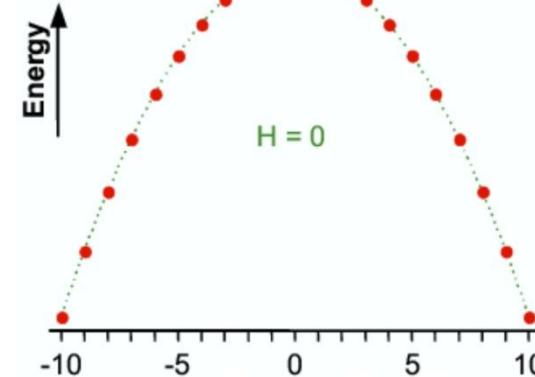
# Magnetic anisotropy :Microscopic origin of single molecule magnet behaviour

## Single Molecule Magnets



12 Mn : 8 Mn<sup>3+</sup> (d<sup>4</sup>), 4 Mn<sup>4+</sup> (d<sup>3</sup>)  
Antiferromagnetically coupled

Spin-orbit coupling :  
projection of the SO states on |S,M<sub>s</sub>>



Zero-Field Splitting:  
Degeneracy lift of the M<sub>s</sub> components of the ground spin state S=10

Strong axial anisotropy : slow relaxation of the magnetization  
Axial anisotropy parameter D<0 (M<sub>s</sub>=±10 ground states)

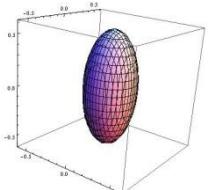
## Usual models for magnetic anisotropy

Giant spin (for the ground state) either for mononuclear or polynuclear systems

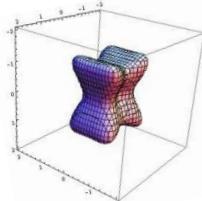
Ising

$$\hat{H} = \underbrace{D\hat{S}_z^2 + E(\hat{S}_x^2 - \hat{S}_y^2)}_{\hat{H} = \hat{S} \overline{D} \hat{S}} + \underbrace{C(\hat{S}_+^4 + \hat{S}_-^4) + D' \hat{O}_4^0 + E' \hat{O}_4^2}_{\text{Higher order terms}}$$

$$\hat{H} = \hat{S} \overline{D} \hat{S}$$



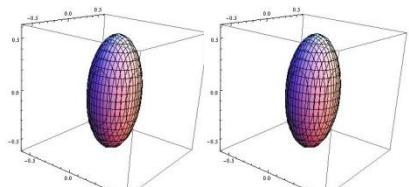
Higher order terms



Multispin Hamiltonian (all the states of the spatial configuration) for polynuclear systems

$$\hat{H} = \sum_i \underbrace{\hat{S}_i \overline{D}_i \hat{S}_i}_{\text{Local anisotropies. An. of the interaction}} + \sum_{\langle i, j \rangle} \underbrace{\hat{S}_i \overline{D}_{ij} \hat{S}_j}_{\text{Magnetic coupling}} + \sum_{\langle i, j \rangle} \underbrace{\bar{d}_{ij} \hat{S}_i \wedge \hat{S}_j}_{\text{Magnetic coupling}}$$

Local anisotropies. An. of the interaction



Magnetic coupling



?

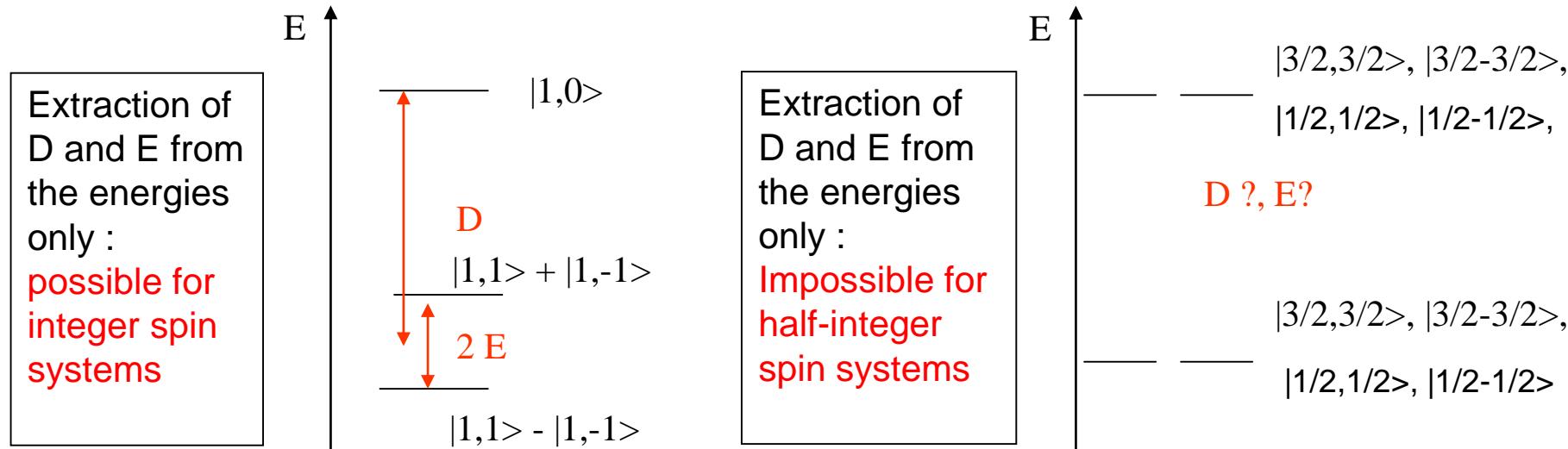
# Extraction of the ZFS tensor and magnetic axes in mononuclear complexes

Phenomenological Hamiltonian :

$$\hat{H}^{\text{model}} = \hat{\mathbf{S}} \cdot \bar{\mathbf{D}} \cdot \hat{\mathbf{S}} = D \left[ \mathbf{S}_z^2 - \frac{1}{3} S(S+1) \hat{\mathbf{I}} \right] + E \left[ \mathbf{S}_x^2 - \mathbf{S}_y^2 \right]$$

$$(\mathbf{D}) = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{pmatrix} \rightarrow (\mathbf{D}) = \begin{pmatrix} D_{xx} & 0 & 0 \\ 0 & D_{yy} & 0 \\ 0 & 0 & D_{zz} \end{pmatrix}$$

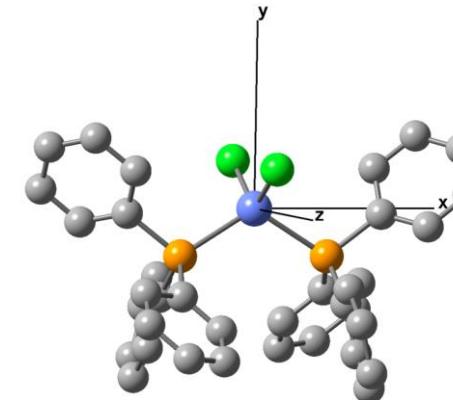
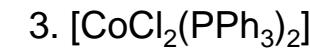
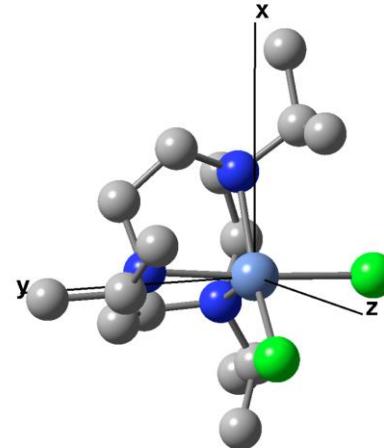
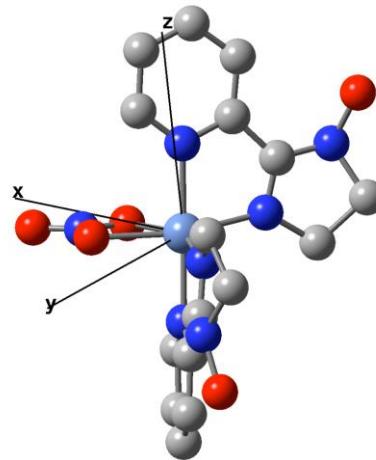
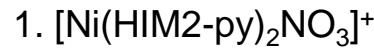
$D = 3/2 D_{zz}$  : axial parameter     $E = 1/2 (D_{xx} - D_{yy})$  : rhombic parameter



**Determination of the effective matrix from both the energies and the wavefunctions**

→ Universal method + extraction of the magnetic axes

## Mono-nuclear Complexes: Methodological Tests

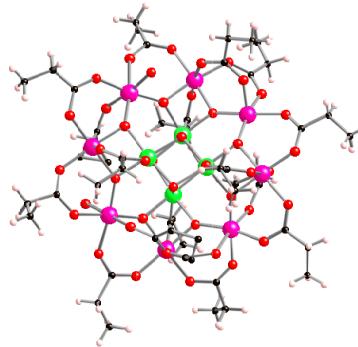


- $\Rightarrow$  Good agreement theory-experiment
- Dynamic correlation plays a non-negligible role
- ZFS dominated by a small number of excited states
- Metal-to-Ligand charge transfer excitations must be taken into account

**Time and energy consuming *ab initio* calculations**

Comp.	ZFS (cm <sup>-1</sup> )	PT2 (12/12)	Exp. (HF-EPR)
1	D	-10.60	-10.15
	E	0.76	0.10
2	D	+16.45	+15.70
	E	3.82	3.40
3	D	-14.84	-14.76
	E	0.54	1.14

# Microscopic origin of magnetic anisotropy

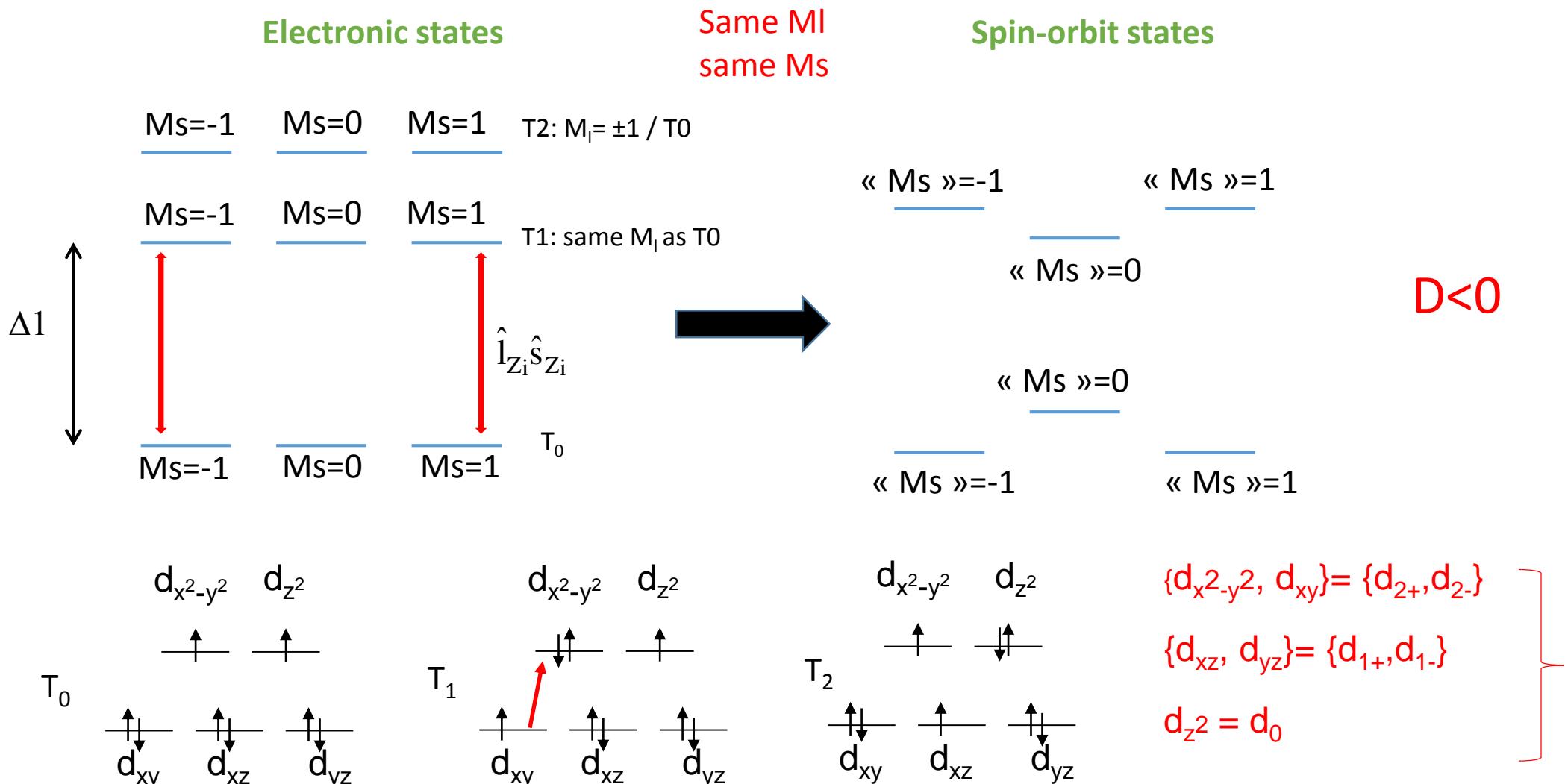


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## Physical origin of magnetic anisotropy : example of Ni<sup>2+</sup>

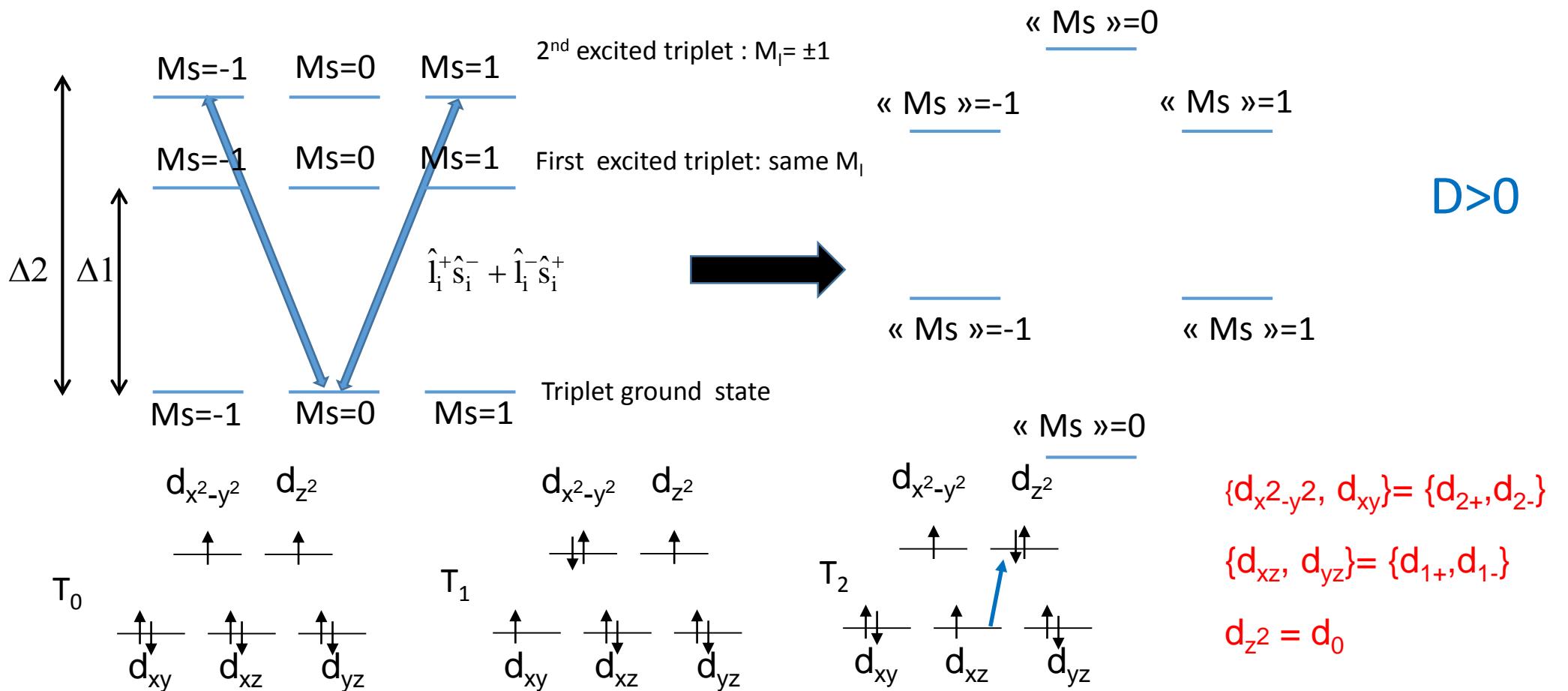
## Relativistic effects : Spin-orbit coupling with excited states

$$\hat{H}_{SO} = \sum_i \zeta_i(r) \hat{l}_i \cdot \hat{s}_i = \sum_i \zeta_i(r) \left( \underbrace{\hat{l}_{X_i} \hat{s}_{X_i} + \hat{l}_{Y_i} \hat{s}_{Y_i}}_{\text{Pairing terms}} + \hat{l}_{Z_i} \hat{s}_{Z_i} \right) = \sum_i \zeta_i(r) \left[ \frac{1}{2} \left( \hat{l}_i^+ \hat{s}_i^- + \hat{l}_i^- \hat{s}_i^+ \right) + \hat{l}_{Z_i} \hat{s}_{Z_i} \right]$$



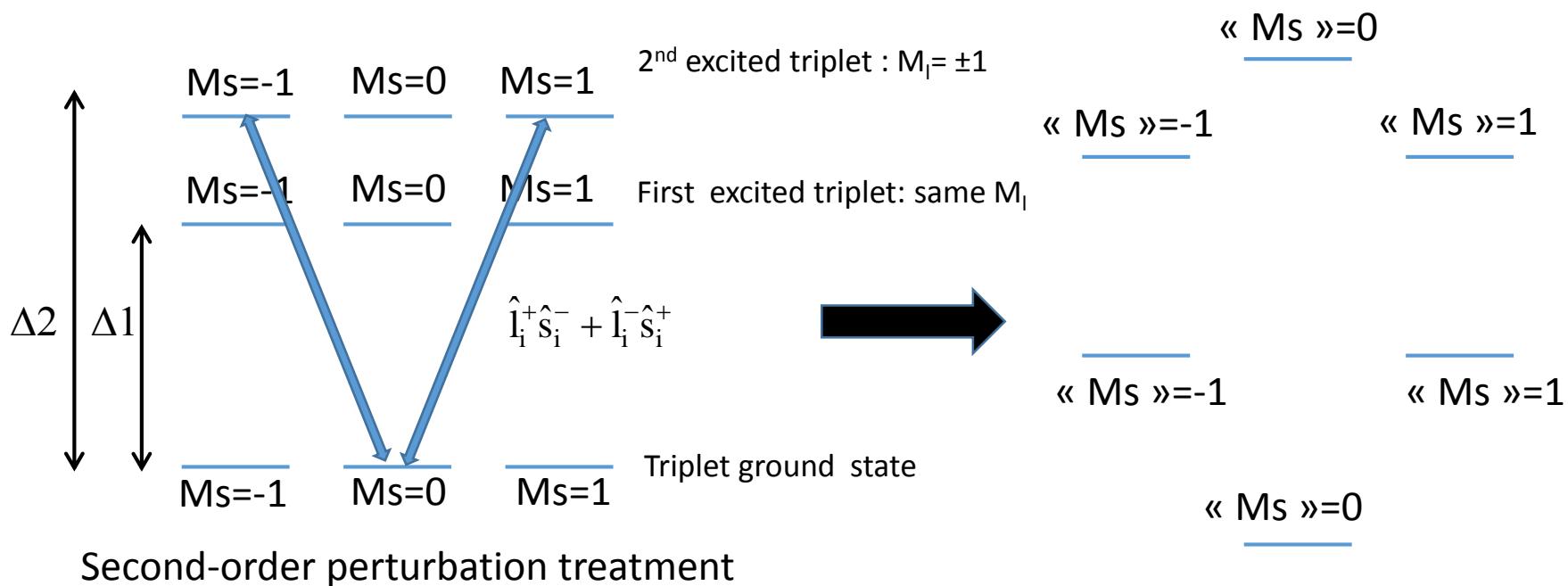
## Relativistic effects : Spin-orbit coupling with excited states

$$\hat{H}_{SO} = \sum_i \zeta_i(r) \hat{l}_i \cdot \hat{s}_i = \sum_i \zeta_i(r) \left( \underbrace{\hat{l}_{x_i} \hat{s}_{x_i} + \hat{l}_{y_i} \hat{s}_{y_i}}_{\text{Electronic states}} + \underbrace{\hat{l}_{z_i} \hat{s}_{z_i}}_{\text{Spin-orbit states}} \right) = \sum_i \zeta_i(r) \left[ \frac{1}{2} \left( \hat{l}_i^+ \hat{s}_i^- + \hat{l}_i^- \hat{s}_i^+ \right) + \hat{l}_{z_i} \hat{s}_{z_i} \right]$$



## Relativistic effects : Spin-orbit coupling with excited states

$$\hat{H}_{SO} = \sum_i \zeta_i(r) \hat{l}_i \cdot \hat{s}_i = \sum_i \zeta_i(r) \left( \underbrace{\hat{l}_{X_i} \hat{s}_{X_i} + \hat{l}_{Y_i} \hat{s}_{Y_i}}_{\text{Electronic states}} + \underbrace{\hat{l}_{Z_i} \hat{s}_{Z_i}}_{\text{Spin-orbit states}} \right) = \sum_i \zeta_i(r) \left[ \frac{1}{2} \left( \hat{l}_i^+ \hat{s}_i^- + \hat{l}_i^- \hat{s}_i^+ \right) + \hat{l}_{Z_i} \hat{s}_{Z_i} \right]$$



$$\langle \phi_I | \hat{H}^{\text{eff}} | \phi_I \rangle = E_i + \sum_{\alpha \notin S} \frac{\langle \phi_I | \hat{H}_{\text{SOC}} | \phi_\alpha \rangle^2}{E_I^0 - E_\alpha^0} \propto \frac{\text{coef.} \zeta^2}{\Delta}$$

$\Delta 1$  versus  $\Delta 2$  ?



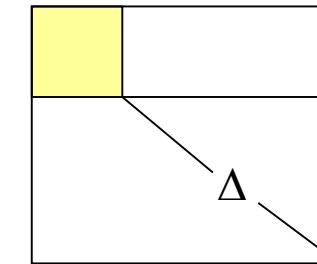
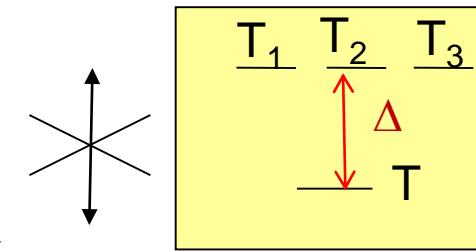
Correlation with Ligand field

# Rationalization : 2<sup>nd</sup> order of perturbation

First test : No ZFS in O<sub>h</sub> symmetry

$$\begin{matrix} T_+ & T_0 & T_- \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & & \end{matrix}$$

$$\begin{matrix} T_1^+ & T_2^+ & T_3^+ & T_1^0 & T_2^0 & T_3^0 & T_1^- & T_2^- & T_3^- \\ i\zeta & 0 & 0 & 0 & \frac{i\zeta}{\sqrt{2}} & \frac{-\zeta}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{i\zeta}{\sqrt{2}} & \frac{-\zeta}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{-i\zeta}{\sqrt{2}} & \frac{\zeta}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & \frac{i\zeta}{\sqrt{2}} & \frac{\zeta}{\sqrt{2}} & -i\zeta & 0 & 0 \end{matrix}$$



$$\begin{matrix} |T_+\rangle & |T_0\rangle & |T_-\rangle \\ \langle T_+| & \frac{-2\zeta^2}{\Delta} & 0 & 0 \\ \langle T_0| & 0 & \frac{-2\zeta^2}{\Delta} & 0 \\ \langle T_-| & 0 & 0 & \frac{-2\zeta^2}{\Delta} \end{matrix}$$

Energy of the first triplet state  $T_{1,2,3,} = \Delta$

$$\langle \phi_I | \hat{H}^{eff} | \phi_J \rangle = \langle \phi_I | \hat{V} | \phi_J \rangle + \sum_{\alpha \notin S} \frac{\langle \phi_I | \hat{V} | \phi_\alpha \rangle \langle \phi_\alpha | \hat{V} | \phi_J \rangle}{E_J^0 - E_\alpha^0}$$

The three components are degenerate

ZFS results from a symmetry lowering + Spin orbit coupling

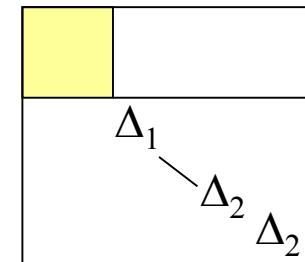
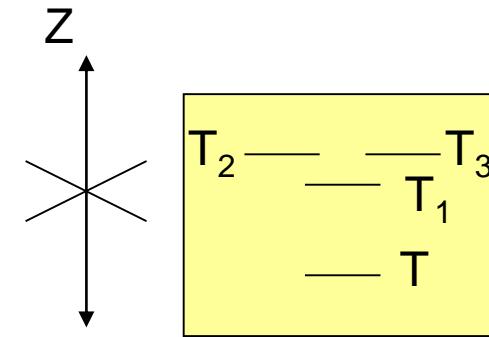
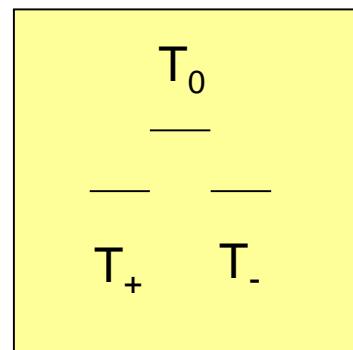
# Rationalization of the D sign : Axial deformation : ZFS in $D_{4h}$ symmetry

$H_{\text{eff}}$  for a  $D_{4h}$  geom. In the magnetic axes frame:

$H_{\text{eff}}$	$ T_+>$	$ T_0>$	$ T_->$
$\langle T_+  $	$-\frac{\zeta^2}{\Delta_1} - \frac{\zeta^2}{\Delta_2}$	0	0
$\langle T_0  $	0	$-\frac{2\zeta^2}{\Delta_1}$	0
$\langle T_-  $	0	0	$-\frac{\zeta^2}{\Delta_1} - \frac{\zeta^2}{\Delta_2}$

Lift of degeneracy « axial »  
between  $M_s=0$  and  $M_s=1$  and  $M_s=-1$

No tunnel splitting



$$\Delta_1 = E(T_1) = \Delta$$

$$\Delta_2 = E(T_2) = \Delta + \frac{35}{4} D_T$$

$$D = -\frac{\zeta^2}{\Delta_1} + \frac{\zeta^2}{\Delta_2}$$

Proposed by Abragam

Elongation along Z :  $\Delta_2 < \Delta_1$        $D < 0$   
Compression along Z :  $\Delta_2 > \Delta_1$        $D > 0$



Rationalization of the D sign

# Rationalization of the rhombic component E : deformation in the (X,Y) plane : ZFS in $D_{2h}$ symmetry

$H_{\text{eff}}$  pour une géométrie  $D_{2h}$  et dans le système d'axes propres

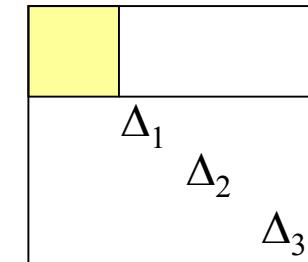
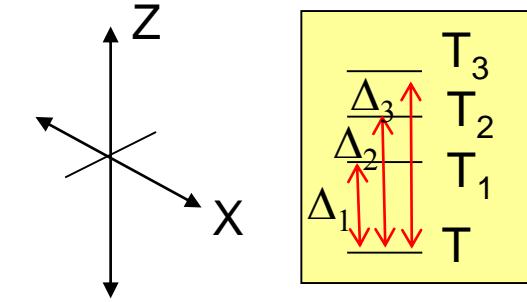
$H_{\text{eff}}$	$ T_+\rangle$	$ T_0\rangle$	$ T_-\rangle$
$\langle T_+  $	$\frac{\zeta^2}{\Delta_1} + \frac{1}{2} \cdot \frac{\zeta^2}{\Delta_2} + \frac{1}{2} \cdot \frac{\zeta^2}{\Delta_3}$	0	$\frac{1}{2} \cdot \frac{\zeta^2}{\Delta_2} - \frac{1}{2} \cdot \frac{\zeta^2}{\Delta_3}$
$\langle T_0  $	0	$\frac{\zeta^2}{\Delta_2} + \frac{\zeta^2}{\Delta_3}$	0
$\langle T_-  $	$\frac{1}{2} \cdot \frac{\zeta^2}{\Delta_2} - \frac{1}{2} \cdot \frac{\zeta^2}{\Delta_3}$	0	$\frac{\zeta^2}{\Delta_1} + \frac{1}{2} \cdot \frac{\zeta^2}{\Delta_2} + \frac{1}{2} \cdot \frac{\zeta^2}{\Delta_3}$

Spectrum

$$\begin{array}{c} T_0 \\ \hline \equiv \\ T_+ \pm T_- \end{array}$$

Axial D and rhombic E  
parameters

$$\left\{ \begin{array}{l} D = -\frac{\zeta^2}{\Delta_1} + \frac{\zeta^2}{2\Delta_2} + \frac{\zeta^2}{2\Delta_3} \\ E = \frac{\zeta^2}{2\Delta_2} - \frac{\zeta^2}{2\Delta_3} \end{array} \right.$$



$$\Delta_1 = E(T_1) = \Delta$$

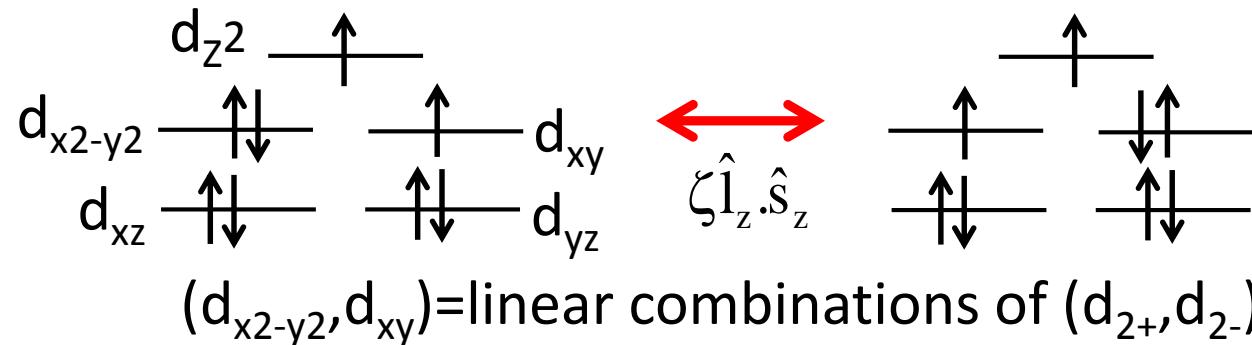
$$\Delta_2 = E(T_2) = \Delta + \frac{35}{4} D_T - D_R$$

$$\Delta_3 = E(T_3) = \Delta + \frac{35}{4} D_T + D_R$$

# Looking for large uniaxial magn. anis. in mononuclear complex

1) Use heavy elements (Important SOC) instead of transition metals

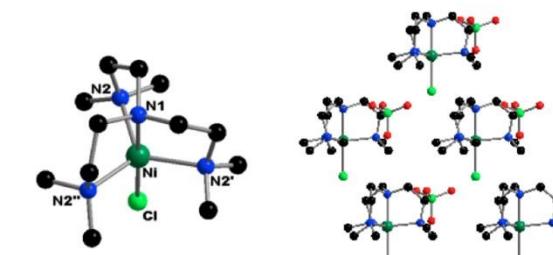
2) Play with first-order SOC: degenerate ground state:



Degenerate ground term components coupled by SOC

→ large splitting of  $M_s = \pm 1$  and  $M_s = 0$  components.

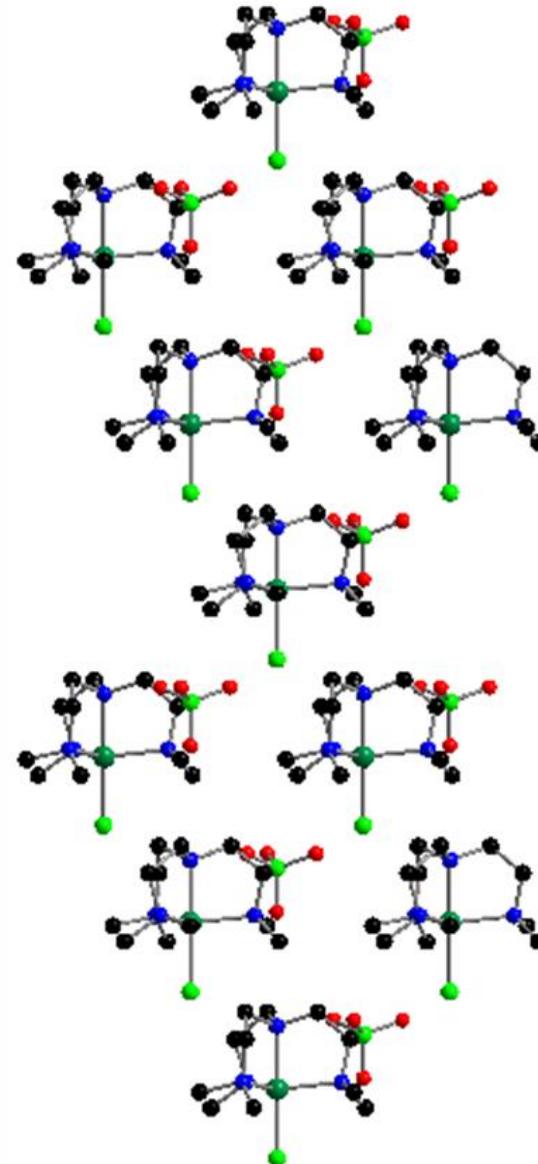
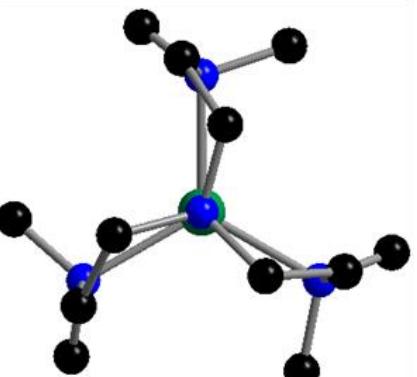
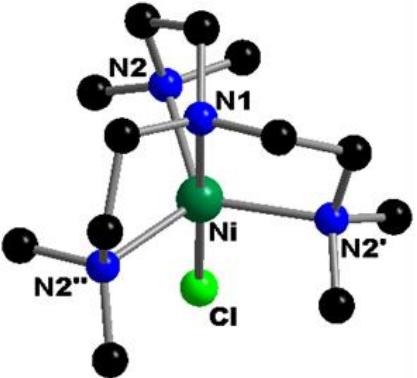
Problem: The Jahn-Teller distortion removes the ground term degeneracy : with exotic coordination (penta- and hepta-coordination).



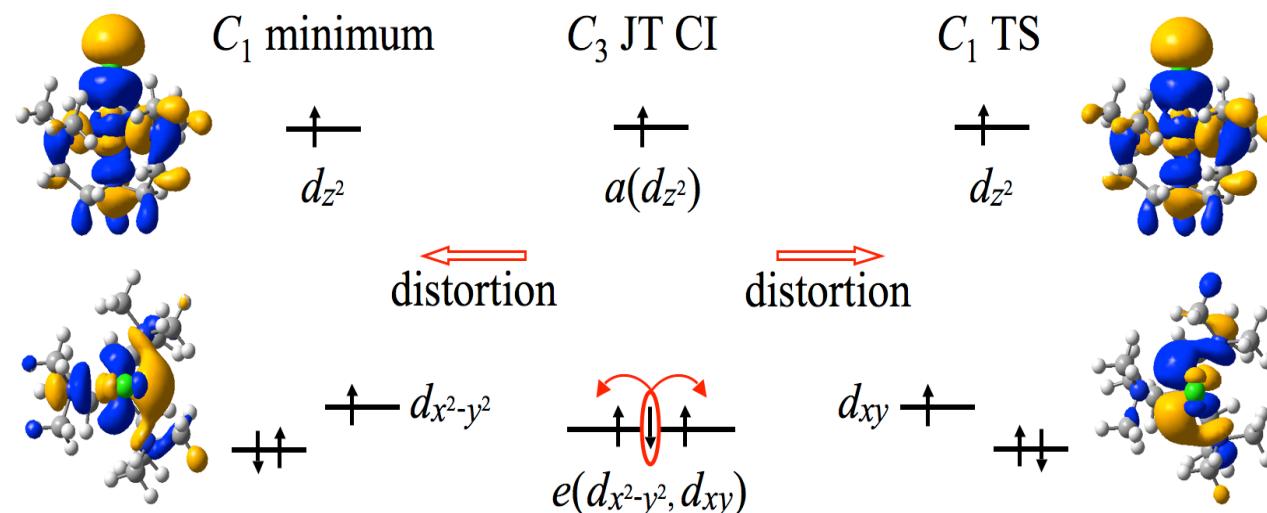
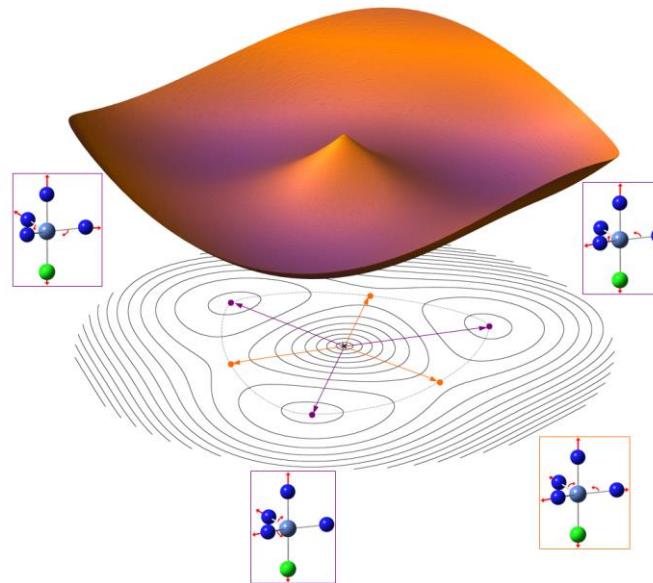
More details in JACS 135 3017-3026 (2013)

## Studied complexes: Cl or Br, ligand

The complex is found of symmetry  $C_3$  experimentally : Means that  $d_{xy}$  and  $d_{x^2-y^2}$  are degenerate !!!



# DFT Results on the Cl compound: minima and saddle point a moat around the conical intersection



Energy barrier including ZPE (well bottom to the saddle point):  $70 \text{ cm}^{-1}$   
This small value rationalizes the X-Ray observation of a  $C_3$  molecule at room temperature

# Modelization of the Spin orbit potential energy curve along the Jahn Teller distortions

Model Hamiltonian matrix describing the interactions resulting from the SOC and the electronic coupling due to the Jahn teller distortions:\*

$$\begin{array}{cccccc}
 & |d_{xy}, -1\rangle & |d_{x^2-y^2}, -1\rangle & |d_{xy}, 0\rangle & |d_{x^2-y^2}, 0\rangle & |d_{xy}, 1\rangle & |d_{x^2-y^2}, 1\rangle \\
 \langle d_{xy}, -1 | & -\Delta + \delta_1 & \delta_2 + i\zeta & 0 & 0 & 0 & 0 \\
 \langle d_{x^2-y^2}, -1 | & \delta_2 - i\zeta & -\Delta - \delta_1 & 0 & 0 & 0 & 0 \\
 \langle d_{xy}, 0 | & 0 & 0 & -\Delta + \delta_1 & \delta_2 & 0 & 0 \\
 \langle d_{x^2-y^2}, 0 | & 0 & 0 & \delta_2 & -\Delta - \delta_1 & 0 & 0 \\
 \langle d_{xy}, 1 | & 0 & 0 & 0 & 0 & -\Delta + \delta_1 & \delta_2 - i\zeta \\
 \langle d_{x^2-y^2}, 1 | & 0 & 0 & 0 & 0 & \delta_2 + i\zeta & -\Delta - \delta_1
 \end{array}$$

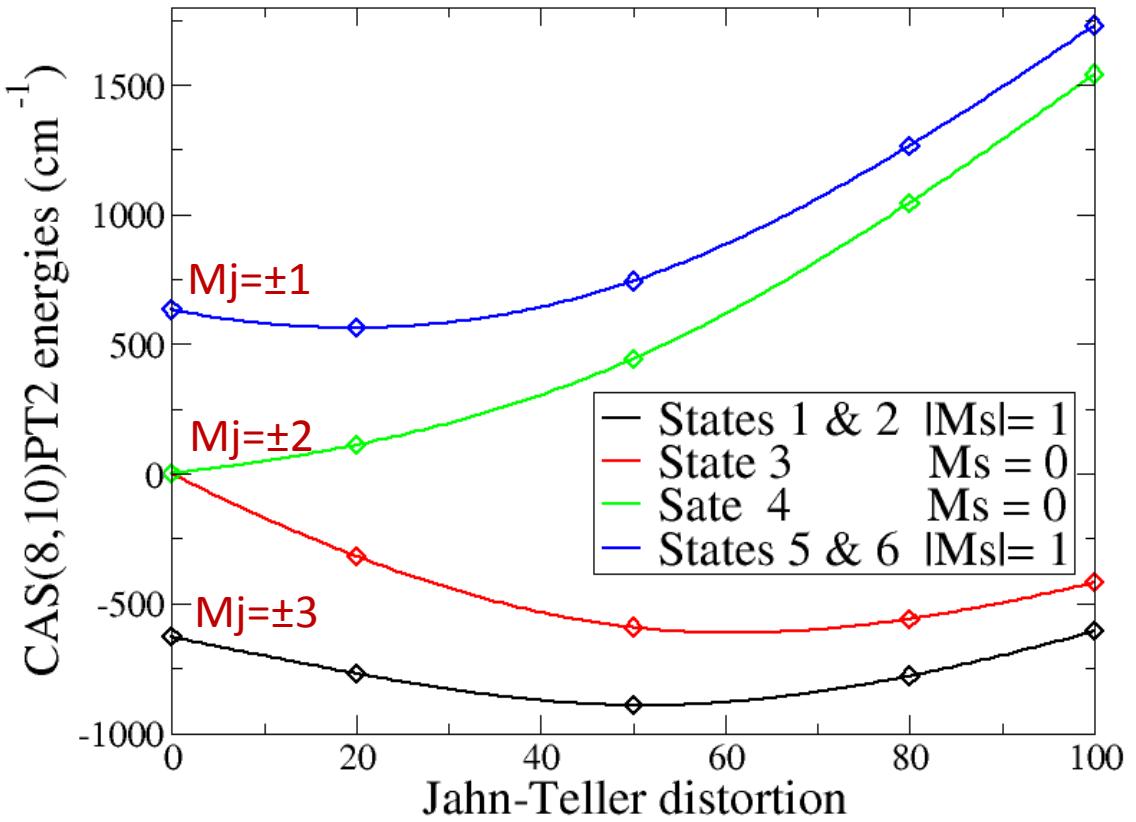
$$\left\{
 \begin{array}{l}
 E_{1,2}(\text{Ms} = \pm 1) = -\Delta - \sqrt{\delta^2 + \zeta^2} \\
 E_{3,4}(\text{Ms} = 0) = -\Delta \pm \delta \\
 E_{5,6}(\text{Ms} = \pm 1) = -\Delta + \sqrt{\delta^2 + \zeta^2}
 \end{array}
 \right.$$



$$\left\{
 \begin{array}{ll}
 E_{1,2}(\text{Ms} = \pm 1) = \mp \zeta; & \text{No distortion} \\
 E_{3,4}(\text{Ms} = 0) = 0 & \\
 \\ 
 E_{1,2,3}(\text{Ms} = 0, \pm 1) = -\Delta - \delta & \text{Strongly} \\
 E_{4,5,6}(\text{Ms} = 0, \pm 1) = -\Delta + \delta & \text{distorted}
 \end{array}
 \right.$$

\*Atanasov; M., et al. *Inorg. Chem.* . 2008, **47**, 8112.

## 2 states SOSI-CASPT2 potential energy curves



Low energy spectrum computed at the two-state SOSI-CAS(8,10)PT2 level of correlation. The  $x$  axis represents a linear distortion between the  $C_3$  (0) and DFT  $C_1$  minimum (100).

- \* Symmetry  $C_3$  : the splitting is around  $635 \text{ cm}^{-1}$ . It is reduced by the distortion.
- \* At this level of calculation, the minimim does not coincide with the DFT minimum, The SO-SI CASPT2 minimum is less distorted .
- \* Note that the ground state is degenerate. One should introduce the second order SOC (other excited states).

## Summary

- \* The reduction of the splitting due to the Jahn-Teller distortion is quite important :  $\Delta E = -635 \text{ cm}^{-1}$  at the  $C_3$  point ;  $-300 \text{ cm}^{-1} < D < -200 \text{ cm}^{-1}$  at the various  $C_1$  studied geometries.
- \* Including higher excited states reduces the values of  $D$  in all considered distorted structures  $-200 \text{ cm}^{-1} < D < -100 \text{ cm}^{-1}$
- \* The better agreement with experiment for the magnetic susceptibility is obtained for the SO-SI CASSCF minimum :  $D = -152 \text{ cm}^{-1}$
- \* This work reports the largest uniaxial magnetic anisotropy observed in mononuclear complex.

HF-HFRPE:  $D_{\text{exp.}}$  between  $-120 \text{ cm}^{-1}$  and  $-180 \text{ cm}^{-1}$

These extremely large values show that the Jahn-Teller distortion is not large enough to completely eliminate the large impact of the spin-orbit coupling between the two lowest states. The rigidity of the ligand preventing strong deformations is probably responsible for this result.

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## Fundings



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